



UNIT 30 QUANTUM MECHANICS: AN INTRODUCTION

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STUDY GUIDE

This Unit consists of three components—the text, an AV sequence and a TV programme. There are fewer pages than in most other Units, and you should find that your study time is somewhat shorter. However, this Unit contains several subtle concepts that normally each take some time to be absorbed. We should also like to point out that the Unit contains some historical background to the development of quantum physics—you do not have to remember *any* of this historical material.

You should work through the AV sequence ‘Wavefunctions of matter’ (Tape 5, Side 1, Band 1), which concerns quantum mechanical waves, when you reach Section 3.2 of the text. The sequence should take you about 40 minutes to study.

The TV programme, ‘Quantum physics—electrons and photons’, can be watched with profit at any stage in your studies of the Unit. Notes for the programme are included in Section 2.

I INTRODUCTION TO THE MODERN PHYSICS BLOCK (UNITS 30–32)

Common sense is something that almost everyone thinks they have in abundance. Although it is a difficult (or perhaps impossible) concept to define, common sense is something that we are often urged to use and to trust.

We have frequently appealed to *your* common sense when we have presented scientific arguments, for example by prefacing apparently uncontentious statements with ‘obviously’ or ‘clearly’. You may have found this irritating, but you would surely agree that *these* two statements are obviously true:

- 1 When someone walks through a doorway without touching the door frame (Figure 1a), the width of the doorway does not affect the walker’s subsequent path.
- 2 When a stone is dropped down a well (Figure 1b), it is possible to predict for certain where the stone will strike the bottom of the well.

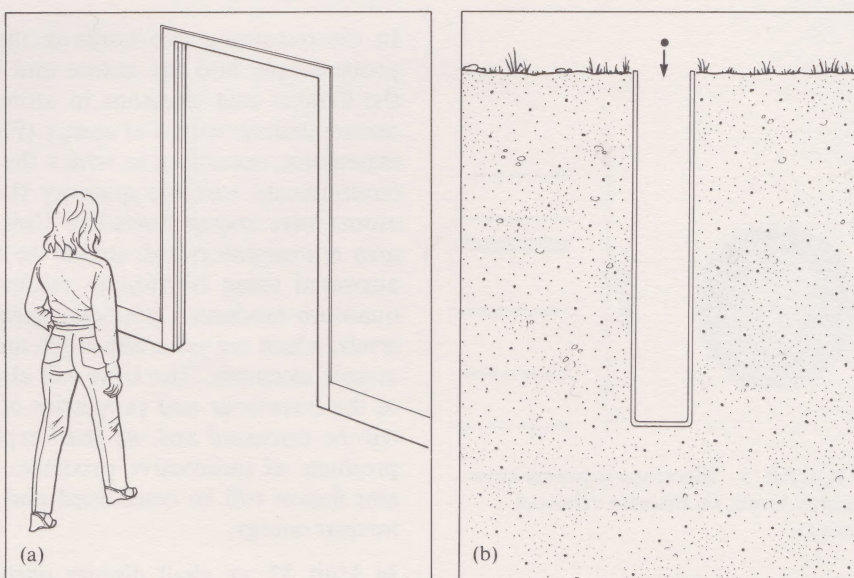


FIGURE 1 Two situations that appear to be amenable to commonsense analyses: (a) someone walking through a doorway; (b) a stone falling down a well.

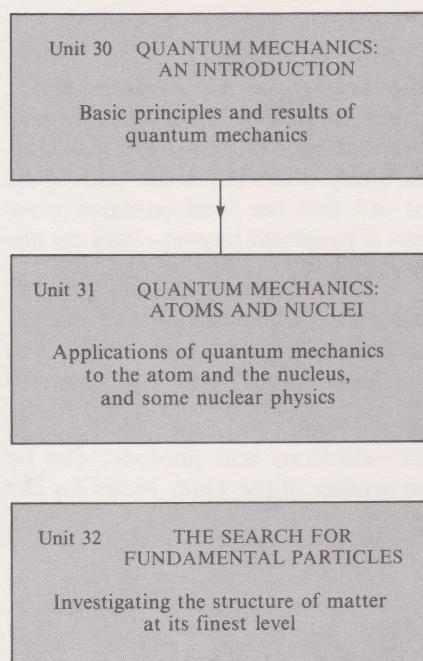


FIGURE 2 The modern physics Block of the Course.

These are uncontentious statements, are they not? Maybe—but they are both believed to be wrong. The theory that implies that the statements are not correct is quantum mechanics, a theory that revolutionized our understanding of the behaviour of matter when it was formulated in the mid-1920s. Until then, motion was believed to be described adequately by the theory of mechanics proposed by Newton almost 250 years before, in 1687 (Unit 3). Newton's laws had given an extremely successful description of matter—the laws could be applied equally well to objects in the everyday world (such as an apple falling from a tree) and to celestial objects (such as the Moon orbiting the Earth). However, soon after the beginning of the 20th century, it was found experimentally that these laws could not account satisfactorily for the behaviour of small-scale matter, such as atoms and subatomic particles. Quantum mechanics does allow this behaviour to be understood, and it can explain how large-scale matter behaves, so this theory has superseded Newtonian mechanics as the best available theory of matter.

This Unit, the first in the modern physics Block (Figure 2), is concerned with some of the most basic concepts and principles of quantum mechanics. It begins by revising the models that are needed to understand the behaviour of electromagnetic radiation (e.g. light), and then goes on to consider the behaviour of matter (electrons, protons, neutrons, etc.). You will see that, contrary to expectations based on everyday experience, each sample of matter has an associated wave. The quantum mechanical interpretation of this wave causes us to abandon the simple commonsense picture of electrons, protons and neutrons, etc., as particles with perfectly defined positions and velocities. According to quantum mechanics, a much more subtle visualization of such microscopic matter is required.

We then move on to discuss one of the most crucial principles of quantum mechanics—the Heisenberg uncertainty principle. This principle radically changed the physicist's view of the concept of measurement. For example, according to Heisenberg, the accuracy with which the *position* of an electron is known unavoidably restricts the accuracy with which its *momentum* can be known simultaneously (and vice versa). This is incomprehensible in terms of Newtonian ideas.

Finally, some of the basic concepts of quantum mechanics are applied to matter in the everyday world in order to see how certain predictions of the theory differ from those based on common sense. When you have completed the Unit, you will have seen that the behaviour of matter is more complicated than it appears to be in everyday life, and you will be able to understand why the two statements made at the beginning of this Introduction are incorrect.

In the remaining two Units in the Block, quantum ideas will be used to probe deeply into the nature and behaviour of matter. You saw earlier in the Course that electrons in atoms have energy levels, that is, they have certain definite values of energy (Figure 3). This is in contrast with everyday experience, according to which the energy of an object is perceived to be a continuously variable quantity (i.e. *not* quantized). Why do electrons in atoms have energy levels? In Unit 31, quantum mechanics will be used to give a straightforward answer to this question, which cannot possibly be answered using Newtonian mechanics. You will also see in Unit 31 that quantum mechanics predicts correctly that atomic *nuclei* also have energy levels, which are generally much more widely spaced in energy than those of atomic electrons. The Unit will also be concerned with some other aspects of the behaviour and properties of nuclei. In particular, radioactive decays will be discussed and we shall explain briefly, in biological terms, why the products of radioactive processes can be hazardous. Also, nuclear fusion and fission will be considered and this will lead us to a brief discussion of nuclear energy.

In Unit 32 we shall discuss particle physics, the branch of physics that concerns the behaviour and interactions of subatomic particles, such as electrons, protons and neutrons. You will see that the structure of matter

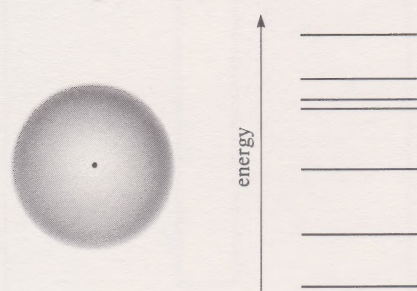


FIGURE 3 Electrons in atoms have energy levels, i.e. discrete values of energy.

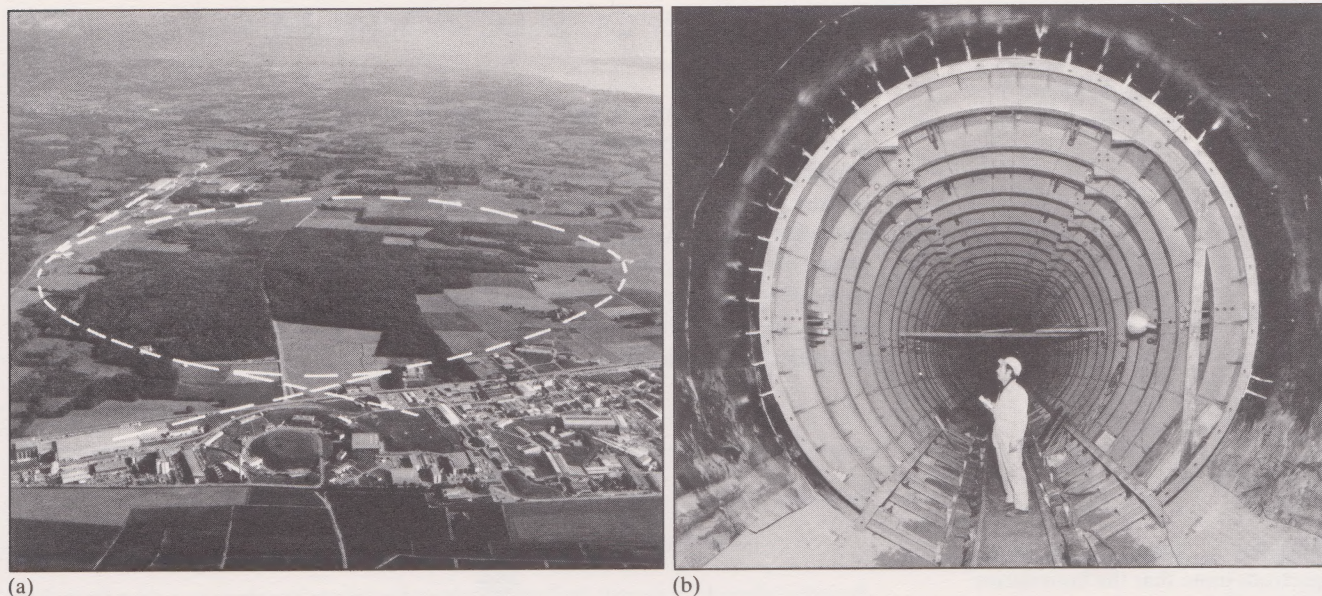


FIGURE 4 (a) The outline of the underground super proton synchrotron (SPS) at Geneva. (b) The tunnel of the SPS when under construction. (Photos courtesy CERN/Science Photo Library)

can be probed at extremely small distances (typically less than 10^{-15} m, a thousand-million-millionth of a metre) using accelerators that produce beams of particles travelling at speeds extremely close to that of light in a vacuum (Figure 4). In order to understand the behaviour of matter that travels at these speeds, it is essential to use the special theory of relativity. The details of this theory will not be considered in this Block, but in Units 31 and 32 some simple results that follow from the theory will be used. In discussions of matter in Unit 30, only matter that is travelling at speeds much less than that of light in a vacuum will be considered, so relativistic complications will not arise.

The results of experiments done with particle accelerators have shown that the most fundamental description of the structure of matter should be given not in terms of atoms and molecules, but in terms of types of fundamental particle known as leptons, quarks and gauge bosons. As you will see in Unit 32, the behaviour of some of these particles transcends everyday experience: some fundamental particles have unusual properties such as 'strangeness' and 'charm'; some can transform spontaneously into other particles; some are detected only indirectly and are expected *never* to be directly observed!

As you can see, we shall be covering some extraordinary material in this Block. When you have finished the three Units, you may well be quite reluctant to use your common sense when you are analysing scientific problems. You may even agree with Albert Einstein, who once said that 'common sense is the deposit of prejudice laid down in the mind before the age of eighteen'.

2 DESCRIBING THE BEHAVIOUR OF ELECTROMAGNETIC RADIATION AND MATTER (TV PROGRAMME)

In this Section, we shall consider the behaviour of electromagnetic radiation and of matter. First, the two models of electromagnetic radiation that you met in Unit 10 are revised, then models of matter (electrons, protons, neutrons, etc.) are examined. The material in this Section constitutes the broadcast notes for the TV programme 'Quantum physics—electrons and photons'.

PHOTON

PHOTOELECTRIC EFFECT

2.1 MODELS OF ELECTROMAGNETIC RADIATION

As you saw in Unit 10, light that can be seen by human beings is a particular type of electromagnetic radiation ('radiation' for short) that has wavelengths between approximately 400 nm and approximately 700 nm (remember that $1 \text{ nm} = 10^{-9} \text{ m}$). There are other types of radiation (for example, γ -rays, X-rays, and microwaves) and each one has a characteristic range of wavelengths (Figure 5). Two entirely different models are required to understand the behaviour of electromagnetic radiation: a wave model, which describes the radiation's propagation, and a particle model, which describes its interactions.

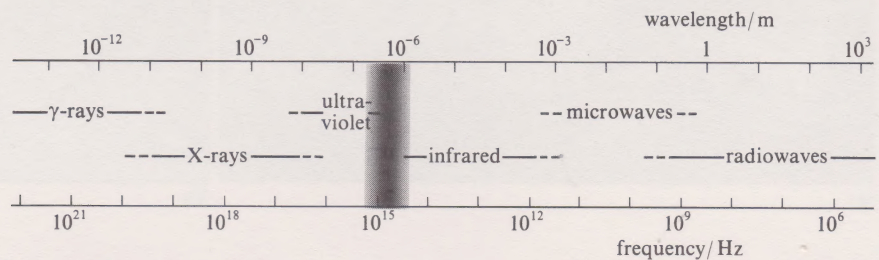


FIGURE 5 The electromagnetic spectrum (note that the boundaries between different regions are not well defined).

2.1.1 WAVE MODEL OF ELECTROMAGNETIC RADIATION

The wave model of radiation is used almost continuously in the analysis of the physics experiment at Summer School. You may remember that in the experiment, a collimated beam of light from a discharge tube propagates towards a diffraction grating, and the diffracted light is observed using an adjustable telescope (Figure 6).

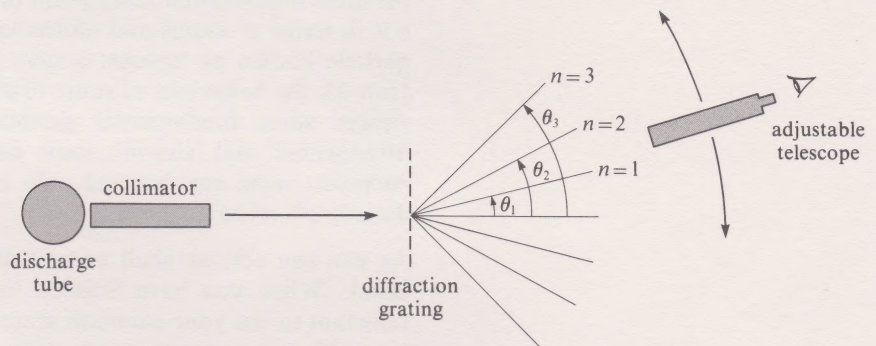


FIGURE 6 When light of wavelength λ impinges on a grating with spacing d , the light is diffracted and the maxima occur at angles θ_n , where $n\lambda = d \sin \theta_n$. This apparatus is used in the physics experiment at Summer School.

For radiation of wavelength λ propagating towards a diffraction grating of spacing d , the angle θ_n of the n th diffracted beam is given by

$$n\lambda = d \sin \theta_n \quad (1)$$

an expression that was derived in Unit 10 using a wave model. For the $n = 1$ beam, Equation 1 says that

$$\lambda = d \sin \theta_1, \quad \text{i.e. } \sin \theta_1 = \frac{\lambda}{d}$$

and for the $n = 2$ beam, Equation 1 says that

$$2\lambda = d \sin \theta_2, \quad \text{i.e. } \sin \theta_2 = 2\left(\frac{\lambda}{d}\right)$$

It is important to remember that diffraction effects are most easily observed (i.e. θ_1 is most easily measurable) if the grating spacing d is not much larger than the wavelength λ of the radiation (Figure 7). If the wavelength is much less than the grating spacing, the diffraction angle θ_1 will be negligibly small—the radiation will appear to propagate straight through (Figure 8).

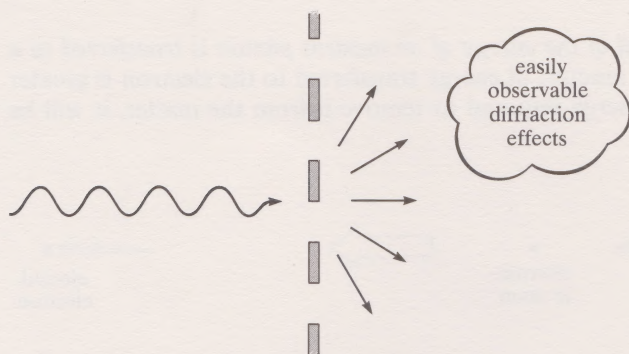


FIGURE 7 Diffraction effects are easily observable if the wavelength of the incident radiation is not much less than the grating spacing.

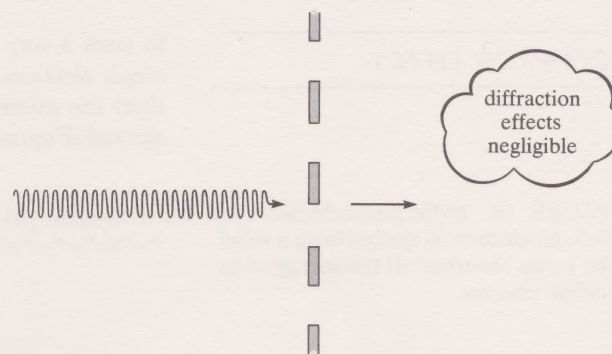


FIGURE 8 If the wavelength is much less than the spacing, diffraction effects will be negligible.

ITQ 1 A beam of red light with wavelength 700 nm ($7.00 \times 10^{-7}\text{ m}$) impinges on a diffraction grating with a spacing of 1400 nm ($1.40 \times 10^{-6}\text{ m}$). Then, in a separate experiment, a beam with the same wavelength is made to impinge on another grating, with a spacing of 1 mm ($1.00 \times 10^{-3}\text{ m}$).

- In which of the two experiments will diffraction be most clearly evident? (Try to answer this without doing any calculations.)
- Calculate the value of θ_1 , the angle of the $n = 1$ diffraction maximum, for each experiment. Check that your answer to this part of the question is consistent with your answer to part (a).

You may remember from Unit 10 that, according to the wave model of electromagnetic radiation, a beam of this radiation is associated with an electromagnetic wave. The frequency f of the electromagnetic wave (in a vacuum) is related to its wavelength λ by the relation

$$f = c/\lambda \quad (2)$$

where c is the speed of light (and of any other type of electromagnetic radiation) in a vacuum, approximately $3.00 \times 10^8\text{ m s}^{-1}$.

2.1.2 PARTICLE MODEL OF ELECTROMAGNETIC RADIATION

When electromagnetic radiation interacts (i.e. exchanges energy) with matter, its behaviour can be described adequately only in terms of a particle model. According to this model, a beam of electromagnetic radiation is considered as a stream of quanta that each have energy and momentum. Each quantum is called a **photon**.

For a beam of radiation with frequency f , each photon has the same energy E given by the Planck–Einstein formula

$$E = hf \quad (3)$$

where h is Planck's constant, the value of which is approximately $6.63 \times 10^{-34}\text{ J s}$. Note that it is easy to state Equation 3 in an alternative form using the expression for the frequency f given in Equation 2:

$$E = \frac{hc}{\lambda} \quad (4)$$

ITQ 2 Consider again the beam of red light of wavelength 700 nm ($7.00 \times 10^{-7}\text{ m}$) described in ITQ 1. What is the energy of each photon in this beam of radiation? (Calculate the answer to two significant figures.)

In Unit 10, you met two types of interaction between radiation and matter—the photoelectric effect and the Compton effect. In the **photoelectric effect**, first explained by Einstein (Figure 9), radiation interacts with matter

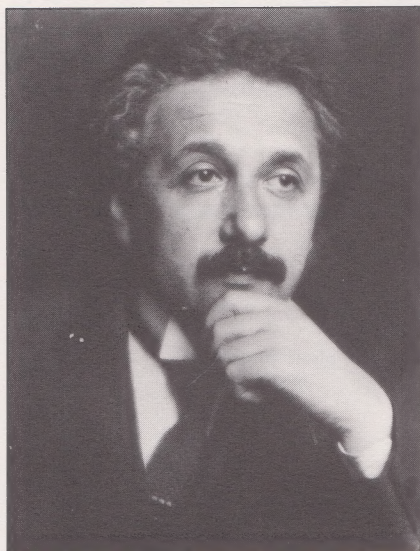
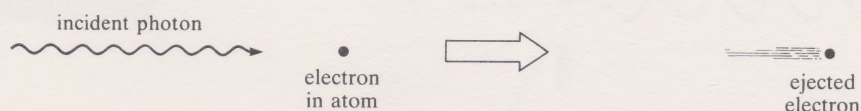


FIGURE 9 Albert Einstein (1879–1955) is widely regarded as the most outstanding scientist of the twentieth century. In January 1919, when he and his first wife divorced, he promised that he would give her the money he would receive when his Nobel Prize was awarded. He was as good as his word: four years later, after he won the 1921 Nobel Prize for physics, he gave her the entire 121 572 Kronor (about \$32 000). He was awarded the Prize for his 'services to theoretical physics, and especially for his discovery of the law of the photoelectric effect'.

COMPTON EFFECT

FIGURE 10 In the photoelectric effect, an electron is ejected from a solid after it has absorbed all the energy of an incident photon.



In the **Compton effect**, radiation is *scattered* by an electron—in this case, *not all of the photon's energy is transferred to the electron*. The effect is illustrated in Figure 11: the incoming photon collides with an electron, which thereby acquires kinetic energy. The photon's energy is therefore reduced.

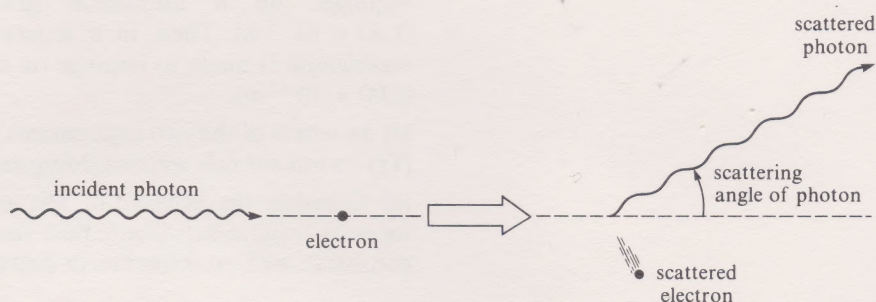
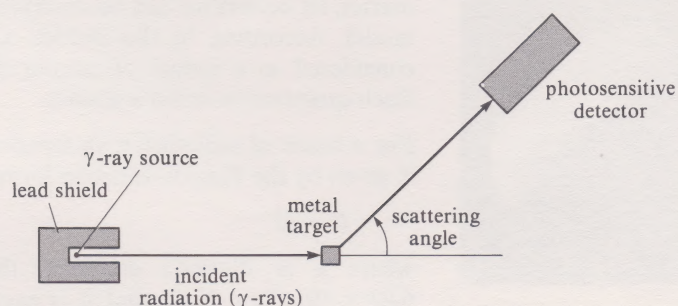


FIGURE 11 In the Compton effect, an electron is scattered by a photon.

- ☐ Is the wavelength of the scattered radiation shorter than, longer than or the same as that of the incident radiation?
- ☒ Longer. Because the energy of the scattered photon is *lower* than that of the incident photon, the wavelength of the scattered radiation must be *higher* than that of the incident radiation. (Remember from Equation 4 that the energy E of a photon is inversely proportional to the wavelength λ of the radiation.)

In the TV programme, we show how the Compton effect can be studied experimentally, using the apparatus illustrated schematically in Figure 12.

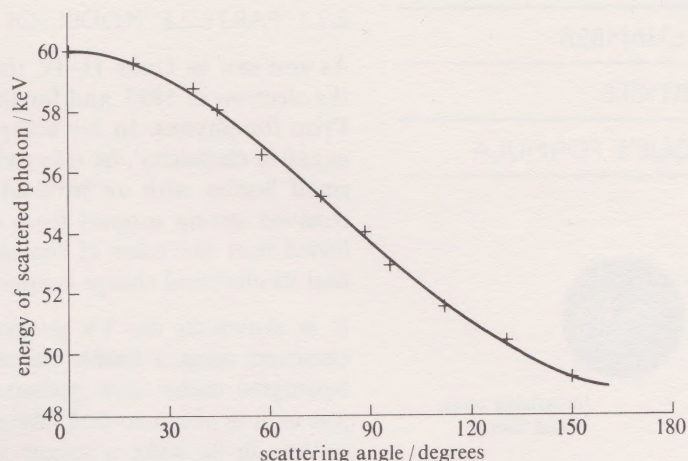
FIGURE 12 A diagram of the apparatus used to demonstrate the Compton effect in the TV programme 'Quantum physics—electrons and photons'.



A collimated beam of γ -rays propagates towards a metal target, and the scattered radiation is detected using a photosensitive detector. The photons are scattered at all angles between zero degrees (the 'straight-through' position, which corresponds to no scattering) and 180° , which is the scattering angle of photons that are reflected directly back in their tracks. At each angle, the scattered photons are found to have a characteristic energy and, moreover, the larger the scattering angle, the lower the energy of the scattered photons that are detected at that angle (Figure 13).

The data can be understood only by assuming that each photon has momentum. As you saw in Unit 10, the momentum of a photon has a direction that is the same as that in which the radiation propagates, and the

FIGURE 13 In the Compton effect, the greater the scattering angle of the incident photon (Figure 12), the more energy it loses. The data in this Figure refer to the experiment demonstrated in the TV programme 'Quantum physics—electrons and photons'.



magnitude p of the photon's momentum is given by

$$p = E/c \quad (5)$$

where E is its energy (remember $E = hf$, Equation 3) and where c is the speed of light in a vacuum. If Equation 5 is used in conjunction with the laws of conservation of energy and momentum, the dependence of a photon's energy loss on its scattering angle can be derived theoretically (we shall not show here how this is done). The theory accounts beautifully for all data on the effect (e.g. Figure 13).

As you saw in Unit 10, the validity of the particle model of light was established by experiments on the Compton effect. These experiments were done by Arthur Compton (Figure 14), who had also been one of the first to formulate a theory of the effect.

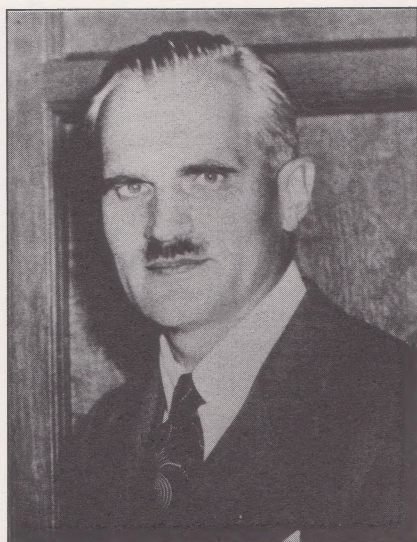


FIGURE 14 Arthur Compton (1892–1962) was awarded a Nobel Prize for physics in 1927 for 'his discovery of the effect named after him'. He did most of this work at the physics department of Washington University in St Louis, Missouri.

SAQ 1 In Table 1, the photoelectric effect is compared with the Compton effect. Fill in the four gaps in the Table.

TABLE 1 Comparison between the photoelectric effect and the Compton effect (SAQ 1)

	Photoelectric effect	Compton effect
type of process: absorption or scattering?		
proportion of the incident photon's energy transferred to the electron		not all of the photon's energy is transferred
law(s) required to understand the process	conservation of energy	

2.2 MODELS OF MATTER

You have just seen that the behaviour of electromagnetic radiation can be described in terms of two models—a wave model, which describes the radiation's propagation, and a particle model, which describes its interactions. But what about the behaviour of *matter*, for example the electron? Surely, a particle model is all that is needed to describe its behaviour?

Remember that in this Unit, we shall consider only matter that is travelling at speeds much less than the speed of light in a vacuum. In this way, the complications of the theory of relativity are avoided.

BUBBLE CHAMBER

FREE PARTICLE

DE BROGLIE'S FORMULA

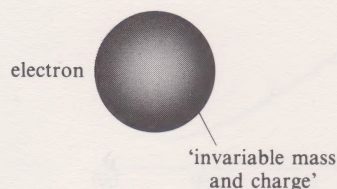


FIGURE 15 According to J. J. Thomson, an electron is a 'small corpuscle with invariable mass and charge'.



FIGURE 16 The track left by an electron in the liquid hydrogen in a bubble chamber. The track is curved because there is a magnetic field perpendicular to the electron's direction of motion.

2.2.1 PARTICLE MODEL OF MATTER

As you saw in Units 11–12, the English physicist J. J. Thomson discovered the electron in 1897, and for this discovery he was awarded the 1906 Nobel Prize for physics. In his acceptance speech, which he entitled 'Carriers of negative electricity', he referred to electrons as 'particles' and as 'corpuscles, small bodies with an invariable mass and charge' (Figure 15). His ideas received strong support from experiments and by 1916 it had been established that the mass of the electron is approximately 9.1×10^{-31} kg and that its electrical charge is approximately -1.6×10^{-19} C.

It is shown in the TV programme that the path of an electron can be observed using a **bubble chamber**. This device contains a liquid (normally hydrogen) under high pressure. If an electron is propelled into the liquid just as it is about to boil, the electron can ionize the atoms it collides with, leaving in its wake a stream of tiny bubbles that can be illuminated and photographed to show the path of the electron (Figure 16). The leaving of a clearly defined track is, of course, just the behaviour you should expect if the electron were a particle, as Thomson envisaged.

The evidence that supports the idea that the electron is a particle is derived from experiments on its *interactions*: for example, its tracks in liquid hydrogen are observable because it interacts with hydrogen atoms. Experiments on other types of matter (e.g. the proton) confirm that their interactions can be understood only if a particle model is used.

So much for the interactions of matter. Let's now move on to discuss the propagation of matter—the motion of matter when it *does not* exchange energy with other matter.

2.2.2 DE BROGLIE'S WAVE MODEL OF MATTER

In September 1923, Prince Louis de Broglie, a young graduate student at the Sorbonne, came up with a truly extraordinary idea. He suggested that the phenomenon of the wave–particle duality might apply not only to radiation—it might also be exhibited by matter. Perhaps, de Broglie proposed, although matter behaves as a particle when it interacts (i.e. exchanges energy with other matter or radiation), it might behave as a *wave* when it propagates (i.e. when it moves without exchanging energy).

Just over a year later, he submitted his doctoral thesis 'Recherches sur la Théorie des Quanta', which contained the details of his theory. The key idea that he presented was that *every sample of matter has an associated wave*. (This wave is *not* an electromagnetic wave—we shall discuss its nature in the AV sequence.) For a particle that is not subject to a net force, that is a **free particle**, he predicted that the wavelength λ_{dB} of the associated wave is given by Planck's constant h divided by the magnitude p of the particle's momentum:

$$\lambda_{dB} = \frac{h}{p} \quad (6)$$

This is known as **de Broglie's formula**. It was de Broglie's innovation to apply this relationship to matter, but the expression also applies to radiation, as you will see in ITQ 3.

ITQ 3 Using Equations 2 ($c = f\lambda$), 3 ($E = hf$) and 5 ($p = E/c$), show that for a photon with momentum of magnitude p the quantity h/p is equal to the wavelength λ of the radiation.

For matter (as opposed to radiation), the magnitude p of its momentum is given by the product of its mass m and its speed v

$$p = mv \quad (7)$$

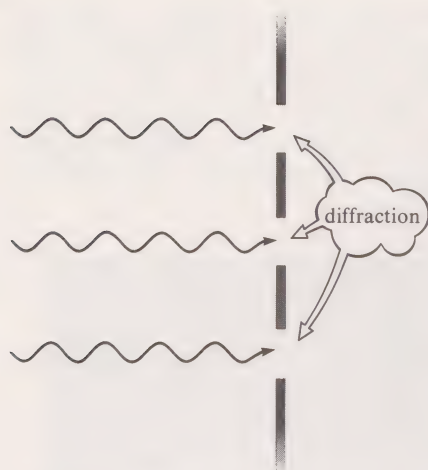


FIGURE 17 Diffraction of matter should be observable if the wavelength of the matter is not much less than the spacing of the grating. (Compare this with Figures 7 and 8.)

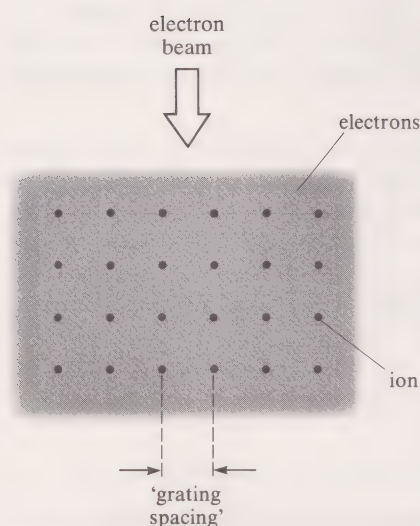


FIGURE 18 A metallic crystal may be regarded as a diffraction grating—the spacing of the ions in the crystal is, loosely speaking, the ‘grating spacing’. This illustration shows only a ‘slice’ through the crystal, which is of course actually three-dimensional.

as you saw in Unit 3. Substitution of Equation 7 into de Broglie’s formula shows that

$$\lambda_{\text{dB}} = \frac{h}{mv} \quad (8)$$

Hence, according to de Broglie, for a free mass m , the faster it is travelling (i.e. the higher its speed v), the shorter is its associated wavelength (i.e. the lower is its value of λ_{dB}).

The committee that examined de Broglie’s thesis praised the originality of his work and duly awarded him a doctorate, but it appears that the examiners did not believe in the physical reality of the newly proposed waves. Other physicists also received the idea with considerable scepticism and, in some cases, derision. In the absence of data to support what was, after all, only a theoretical conjecture, the idea could be ridiculed with impunity. Louis de Broglie was regarded by many as an eccentric and by some as a crank, but he was taken seriously by Einstein, who stressed the importance of doing experiments to investigate the validity of de Broglie’s idea.

How could the idea be checked experimentally? In order to answer this question, think again about the propagation of electromagnetic radiation.

- ☐ Which type of experiment involving radiation shows that its propagation can be understood only by using a *wave* model?
- ☒ Diffraction experiments. In these experiments, radiation propagates towards a grating whose spacing is not much greater than the wavelength of the radiation (Figure 17).

If each sample of free matter does have an associated wave, then such matter should be noticeably diffracted by a grating with a spacing that is of roughly the same order of magnitude as the matter’s wavelength (de Broglie actually pointed this out to the examiners of his thesis!). Consider, for example, a free electron that has been accelerated by a few hundred volts; such an electron has a de Broglie wavelength of approximately 10^{-10} m. Gratings that should noticeably diffract this electron are available in the form of pure metallic crystals which, as you saw in Units 13–14, can be visualized as a regular array of ions in a pool of electrons (Figure 18). Such a structure is closely analogous to that of a conventional optical grating, whose opaque and transparent parts correspond respectively to the ions and the electrons. The average spacing between the ions is, loosely speaking, the ‘grating spacing’ of the crystal.

The question of whether electrons can be diffracted by metallic crystals was investigated in 1925 by Clinton Davisson and his junior collaborator Lester Germer at the Bell Telephone Laboratories in New York. In 1928, G. P. Thomson (the son of J. J. Thomson) independently investigated the question at the University of Aberdeen. The apparatus used by Thomson was similar to the one shown schematically in Figure 19: electrons are accelerated through several hundred volts, propagate through a target of metal foil and are then detected.

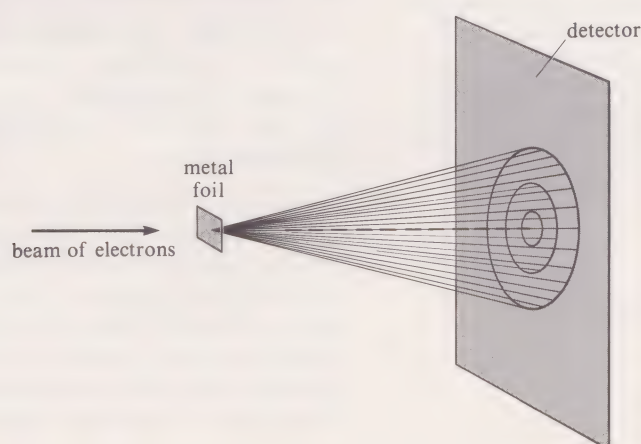


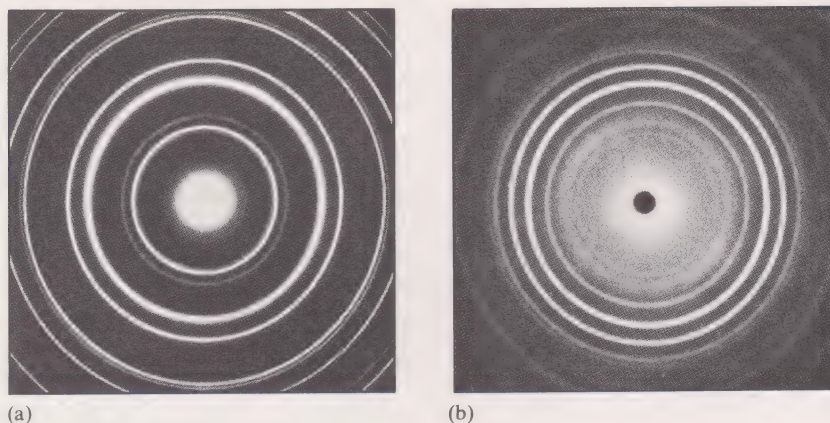
FIGURE 19 Schematic diagram of the type of apparatus used by G. P. Thomson when he observed the diffraction of electrons.

WAVE-PARTICLE DUALITY

QUANTUM

QUANTUM MECHANICS

FIGURE 20 (a) A diffraction pattern obtained using a beam of electrons and a target of gold crystals, in the configuration shown in Figure 19. (b) A diffraction pattern obtained using a beam of X-rays and a target of zirconium oxide crystals, also in the configuration shown in Figure 19.



Thus, de Broglie's revolutionary idea that a wave is associated with each sample of free matter was vindicated, at least for electrons (it was later checked that the idea applied equally well to other types of matter, e.g. protons, neutrons, nuclei, atoms and molecules). The conclusion was inescapable: the idea of **wave-particle duality** applies both to electromagnetic radiation *and* matter.

Louis de Broglie (Figure 21) was subsequently awarded the 1929 Nobel Prize for physics and, eight years later, the Prize was shared by Davisson and G. P. Thomson. The Thomson family certainly did make diverse contributions to our understanding of the electron. G. P. Thomson's Nobel Prize was awarded for his verification of the *wave* model of the electron, whereas his father, J. J. Thomson, had received his Prize for demonstrating that the electron behaves as a *particle*!



FIGURE 21 Prince Louis de Broglie (1892–1987) was of noble descent, his ancestors having served French monarchs since the time of Louis XIV. He was awarded the 1929 Nobel Prize for physics for 'his discovery of the wave nature of electrons'.

2.3 QUANTA AND QUANTUM MECHANICS

You have now seen that there is an analogy between the behaviour of electromagnetic radiation and that of matter. A particle model is required to describe their interactions, whereas a wave model is needed to describe their propagation. Hence, there is a problem of terminology: it is not strictly correct to describe matter (electrons, protons, neutrons, etc.) and electromagnetic radiation simply as 'particles' or as 'waves'. These terms each apply only in certain circumstances ('particles' for their interactions, 'waves' for their propagation).

It is therefore convenient to apply the term **quantum** to radiation *and* matter—by definition, a quantum behaves as a particle in interactions and as a wave in propagation. The advantage of using this term is that it avoids the need separately to specify whether propagation or an interaction is being described.

Although it is best to refer to electrons, protons, neutrons, etc., as quanta, it must be admitted that the use of the term to describe matter is somewhat unconventional. Scientists nearly always refer to electrons, protons, neutrons, etc., as particles, which of course should strictly speaking be used only to describe them when they interact. In the remaining Units of the Course, we shall normally follow this widely accepted convention, and we hope that you will tolerate this laxity. However, in this Unit we shall continue to use the 'quantum' terminology.

You may already have wondered whether there exists a theory that can describe both the propagation *and* interactions (i.e. wave and particle behaviour) of matter, just as the theory of quantum electrodynamics can describe the propagation and interaction of radiation (Unit 10). Such a theory has been formulated—it is called **quantum mechanics**. Sometimes the term *quantum theory* is used as a synonym for quantum mechanics, but we shall use the former term in a more general sense to encompass all the theoretical concepts that apply to all quanta (i.e. matter *and* radiation).

In the remainder of this Unit, some of the basic concepts of quantum mechanics will be described and then, in Unit 31, they will be applied to the atom and the nucleus.

SUMMARY OF SECTION 2

- 1 The propagation of electromagnetic radiation is described by a wave model, which enables the diffraction of radiation to be understood.
- 2 The interactions of electromagnetic radiation with matter can be described by a particle model in which each quantum of radiation (photon) has an energy $E = hf$ and a momentum of magnitude $p = E/c$, in the usual notation. This model enables the photoelectric effect and the Compton effect to be understood.
- 3 The propagation of free matter is described by de Broglie's-wave model, according to which free matter has an associated wavelength $\lambda_{dB} = h/p$, in the usual notation. This model successfully predicted that electrons can be diffracted.
- 4 The interactions of matter can be understood using a particle model.
- 5 The idea of wave-particle duality applies to both electromagnetic radiation and matter.
- 6 Quantum mechanics is a theory that describes the wave and particle behaviour of matter.

SAQ 2 (a) A student doing the physics experiment at Summer School arranges for a collimated beam of sodium light with a wavelength of 589 nm to impinge perpendicularly on a diffraction grating whose spacing is 1 670 nm. At what angle will the $n = 1$ diffraction maximum be observed?

(b) At what angle would the $n = 1$ diffraction maximum have occurred if the student had used, instead of light, a collimated beam of free electrons with a speed of $1.00 \times 10^6 \text{ m s}^{-1}$? (Take the mass of the electron as $9.11 \times 10^{-31} \text{ kg}$, and remember that $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$.)

SAQ 3 A beam of X-rays has a frequency of 10^{18} Hz . (Remember that $1 \text{ Hz} = 1 \text{ s}^{-1}$).

- (a) What is the energy of each X-ray photon?
- (b) What is the momentum of each X-ray photon?

SAQ 4 Fill in the two missing bits of evidence in Table 2 (*overleaf*).

SAQ 5 Which *two* of the following statements about quantum mechanics are correct?

- (a) It can describe the propagation of visible light.
- (b) It can describe the interactions of electrons.
- (c) It can describe the propagation of electrons.
- (d) It applies only to electrons.

TABLE 2 Summary of experimental evidence for the particle and wave models of electrons and electromagnetic radiation (SAQ 4)

Model	Experimental evidence
particle model of the electron	☐ the electron has a definite charge and mass ☐ an electron can leave a clearly defined track in a bubble chamber
wave model of the electron	☐
particle model of electromagnetic radiation	☐ photoelectric effect ☐
wave model of electromagnetic radiation	☐ diffraction (e.g. the physics experiment at Summer School)

3 QUANTUM MECHANICAL DESCRIPTIONS OF MATTER

In this Section, the extraordinary nature of the type of wave associated with matter will be described. Then, in the AV sequence 'Wavefunctions of matter', the waves associated with matter in different circumstances (free and confined) will be illustrated and discussed.

3.1 WHAT TYPE OF WAVE IS ASSOCIATED WITH MATTER?

Consider the experiment shown schematically in Figure 22: a beam of free electrons (i.e. matter) impinges on a single slit and a diffraction pattern is observed on a screen, which serves as a detector. This is precisely analogous to the observation that light is diffracted when it is shone on a single slit (Unit 10).

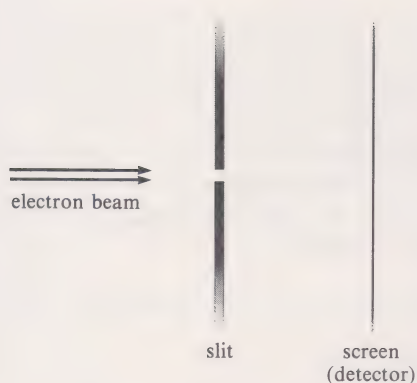


FIGURE 22 A beam of electrons impinges on a single slit. Diffracted electrons are subsequently detected on a screen.

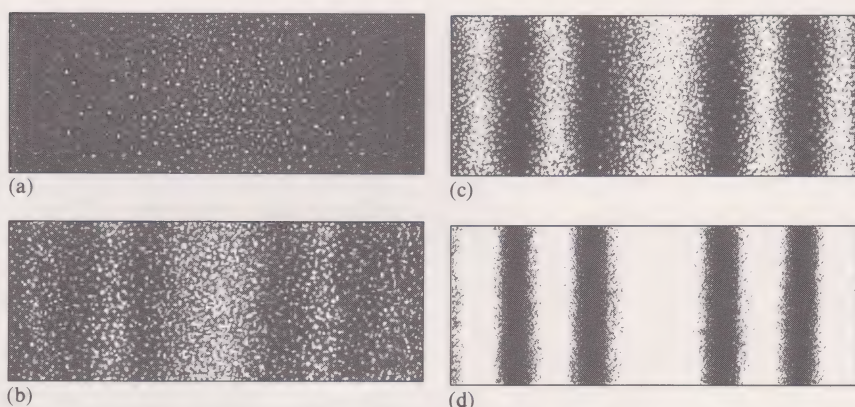


FIGURE 23 As more electrons arrive at the screen (Figure 22), a diffraction pattern gradually builds up.

Each electron is observed to arrive at a definite point on the screen, and, as more electrons are detected, the diffraction pattern builds up. When only a few electrons have arrived, a pattern is scarcely observable (Figure 23a), but it gradually becomes easier to discern (Figures 23b–d) until eventually a practically smooth distribution is observed. This distribution can be represented conveniently by a graph of the intensity of the pattern plotted against diffraction angle (Figure 24). The intensity in a given small region represents the number of electrons that are detected in that region in a given time.

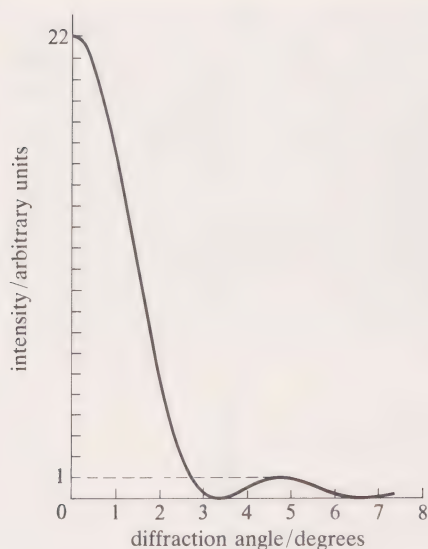


FIGURE 24 The intensity of the diffraction pattern plotted against diffraction angle (relative to the straight-through position).

In Figure 24, you can see that the number of electrons that arrive at a small area around the straight-through position (zero diffraction angle) is approximately 22 times the number that arrive at a small area around the first diffraction maximum (where the diffraction angle is approximately 4.8°). Notice that *no* electrons are detected at the diffraction *minima*.

The observation that each individual electron arrives at a definite point somewhere on the screen can be understood using a *particle* model. However, the shape of the diffraction pattern can be predicted successfully only by using de Broglie's *wave* model, according to which the positions of the diffraction maxima should depend only on the wavelength of the beam (remember, $\lambda_{dB} = h/p$) and on the width of the slit.

So far so good; you will probably not have been surprised by this description of electron diffraction. But let us now consider more closely the behaviour of an individual electron in the beam. Is it possible to predict where the electron will *not* be detected? The answer is Yes—the wave model of electrons can be used to predict the positions of the diffraction minima, where no electrons arrive.

Now another question: is it possible to predict for certain where an individual electron *will* be detected? Believe it or not, the answer is No—it is *absolutely impossible to predict for certain where a particular electron will be detected*. It is possible to predict only the relative probability that the electron will arrive in a particular region of the detector. For example, in the case depicted in Figure 24, the electron is approximately 22 times as likely to be detected in an area opposite the slit (zero diffraction angle) as it is to be detected in an area of the same size around the first diffraction maximum (a diffraction angle of about 4.8°).

ITQ 4 How much more likely is an electron to be detected in a small area around the straight-through position (zero diffraction angle) than it is to be detected in an area of the same size around the diffraction angle of 2° ?

Just as it is not possible to predict for certain where the electron will be detected, so it would be impossible to predict where any other type of matter would be detected after it had been diffracted by the slit.

Hence, a physicist who is asked to predict the behaviour of a single quantum of matter is in rather a similar position to that of opinion pollsters before an election. The pollsters can forecast with reasonable accuracy the percentage of an electorate that will vote for each party, but they cannot predict with certainty the party for which a randomly selected individual will vote (without actually asking!).

All this talk of being able to predict only the odds on the outcome of the diffraction of a single quantum may well strike you as very strange. Isn't physics an exact science? Let us try to anticipate your objections to this unpredictability: consider the following argument put forward by a sceptic.

The nature of the diffraction experiment you have described has not been specified carefully enough. All you have said is that an electron passes *some-where* through the slit. If an extremely careful study were made of the incident electron, its original path could be determined precisely and this determination would enable you to identify the atom (or atoms) in the rim of slit with which the electron interacts. It should then be possible to calculate (in principle, at least) precisely how the electron is scattered from the atom (or atoms), and hence it should be possible to determine the direction in which the scattered electron travels. That would enable you to calculate *exactly* where the electron strikes the detector.

This commonsense argument is based on the crucial assumption that *it is possible to determine precisely where the electron enters the slit and, at the same time, its direction of motion*.

Terms in AV Sequence

WAVEFUNCTION

STANDING WAVE

 SCHRÖDINGER
EQUATION

How could this assumption be checked experimentally? Let us begin by trying to specify very accurately the position of the electron as it passes through the slit. We shall use the apparatus shown in Figure 25, in which the electrons emerge from a hole H (in a container) positioned right up against the slit S. If the hole were extremely small, we should know very accurately *where* the electron enters the slit. But if the hole were extremely small, *the electron could be significantly diffracted by the hole before it enters the slit*. Hence, we should not know accurately the *direction* of motion of the electron as it enters the slit. It would therefore be impossible to use the laws of conservation of energy and momentum to work out the outcome of the collision of this electron with any particular atom (or atoms) in the slit.

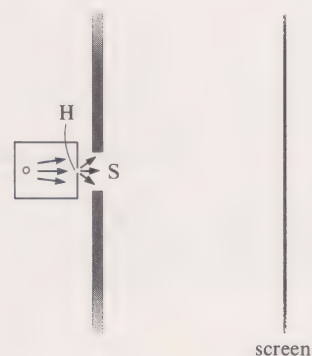


FIGURE 25 A configuration of the apparatus shown in Figure 22: electrons emerge from a tiny hole H that is close to the single slit S.

So much for an attempt to fix accurately the *position* of the electron before it enters the slit. Let us now adopt a different strategy and try to fix its *direction*, using *only* the apparatus shown schematically in Figure 26, in which the slit S is very far away from the hole H. Now, because the slit subtends such a small angle with respect to the centre of the hole H, the direction of motion of the electron as it passes through the slit is defined quite accurately (Figure 26). Unfortunately, because the electron's direction is specified so accurately, our knowledge of the position of the electron as it passes through S has had to be sacrificed. It is not known whether the atom (or atoms) at the rim of the slit is (or are) being struck a glancing blow or a direct hit, or indeed which atom (or atoms) is (or are) being struck. Once again, the laws of conservation of energy and momentum cannot be used to find the path of the electron after it emerges from the slit.



FIGURE 26 Another configuration of the apparatus shown in Figure 22: electrons emerge from a hole H that is very distant from the single slit S.

We hope that you are now convinced that it is *not* possible to predict with certainty where an *individual* electron will be detected after it has been diffracted. It is possible to predict accurately only the diffraction pattern produced by a *very large number* of electrons, and this pattern can be used to calculate the *probabilities* of an individual electron's arrival in different regions of the detector. This conclusion applies equally well to other types of matter, e.g. protons, neutrons, atoms and molecules, when they are diffracted.

Now let us return to the problem of specifying the nature of the wave associated with each sample of matter. You saw in Unit 10 that the wavelength of a light wave and the width of the illuminated slit determine the

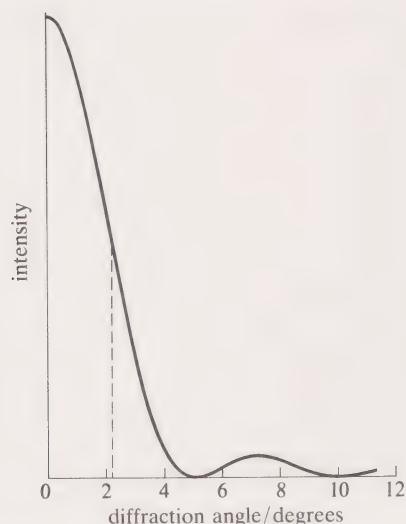


FIGURE 27 See SAQ 6.

optical diffraction pattern. You have now seen that the pattern produced by the diffraction of matter determines the *probabilities* that a quantum of matter will arrive at given regions of the detector. It should therefore be plausible to you that

the wave associated with a quantum of matter is in turn associated in some way with the relative probabilities of detecting the quantum in different regions of space.

In Section 3.2, we shall discuss further the nature of the wave associated with each quantum of matter and we shall show how such waves can be visualized.

SAQ 6 A beam of free protons propagates towards a single slit, and a diffraction pattern is detected behind the slit (the apparatus is very similar to the one shown in Figure 22). The intensity of the diffraction pattern that is observed (after many protons have been detected) is shown in Figure 27.

(a) In which region of the screen is an individual proton in the beam *most* likely to be detected?

(b) In which region(s) of the screen will an individual proton definitely *not* be detected?

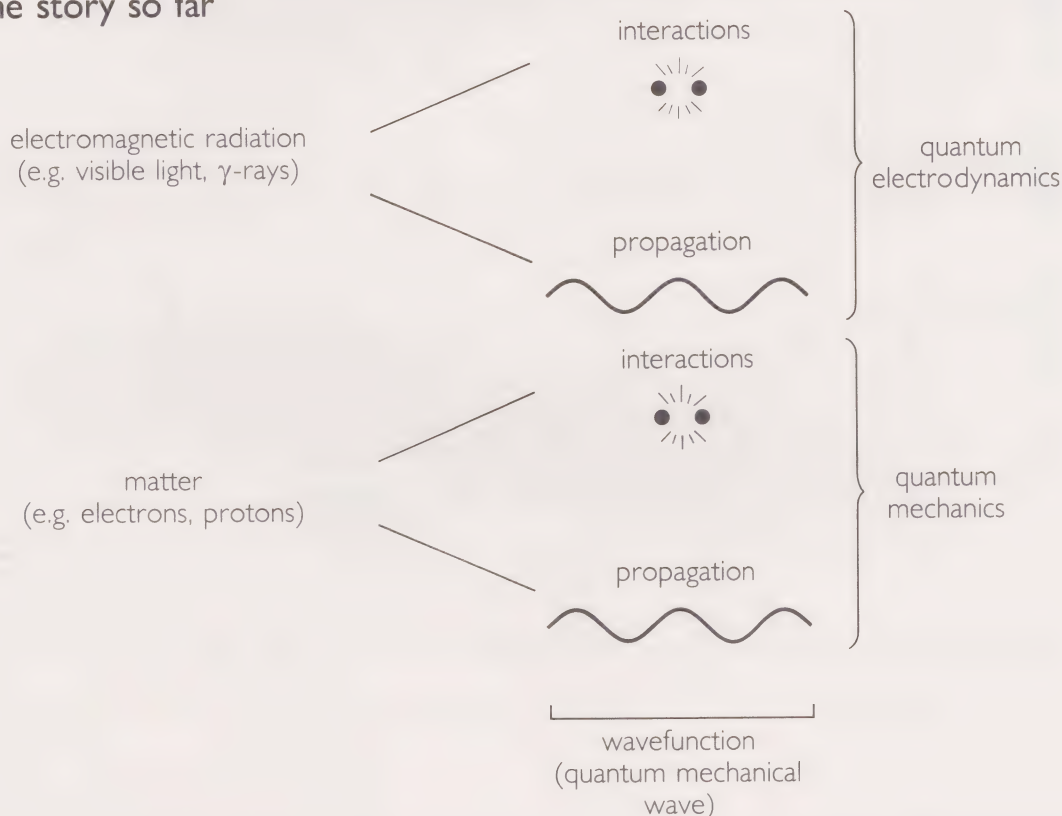
(c) What is the probability that an individual proton will be observed with a diffraction angle of 2.2° (relative to the probability that it will be observed with zero diffraction angle)?

3.2 WAVEFUNCTIONS (AV SEQUENCE)

You should now study the AV sequence 'Wavefunctions of matter' on Tape 5, Side 1, Band 1, then have a go at SAQs 7–9.



The story so far



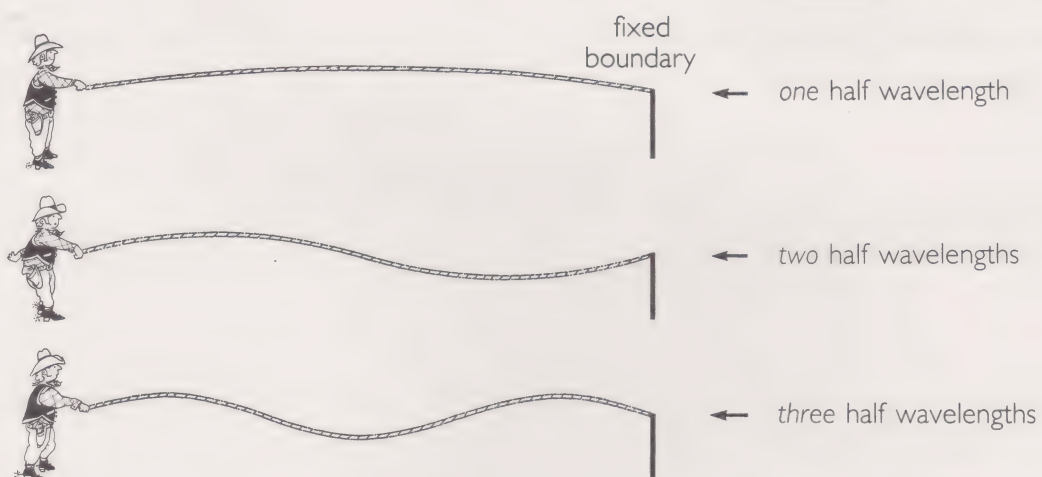
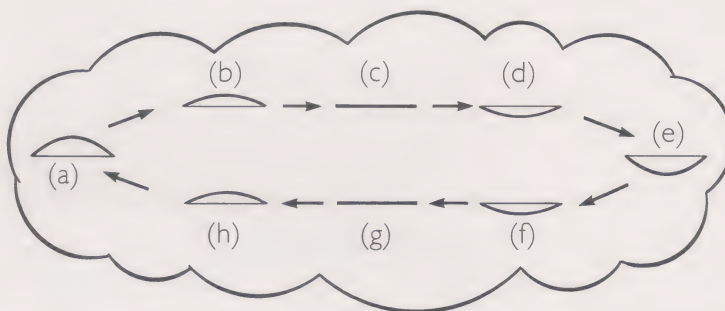
2 Two types of wave on a rope

(i) Infinite sine waves



Sine waves of any wavelength can propagate along an infinitely long rope.

(ii) Standing waves



Each standing wave has a whole number of half-wavelengths between its fixed ends.

3 Two types of wavefunction

(i) Infinite sine waves as wavefunctions

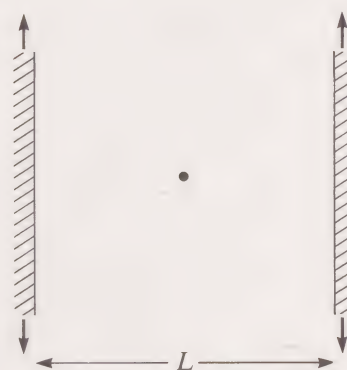


associated with a *free* quantum
(i.e. a quantum on which zero net
force acts)

(ii) Standing waves as wavefunctions



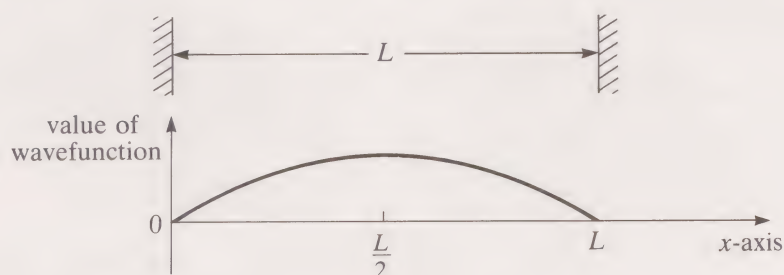
associated with a
quantum that is *confined*
between two parallel,
impenetrable reflecting
plates that are infinitely
high, separated by
distance L



What do these wavefunctions mean?

4 Interpreting wavefunctions

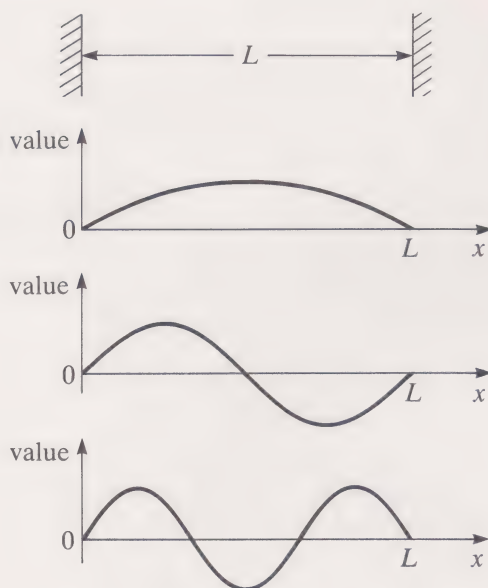
- difficult to discuss rigorously
- we shall consider only the interpretation of wavefunctions that are standing waves



The probability of detecting a quantum of matter in a given small region of space is proportional to the square of the value of the wavefunction in that region.

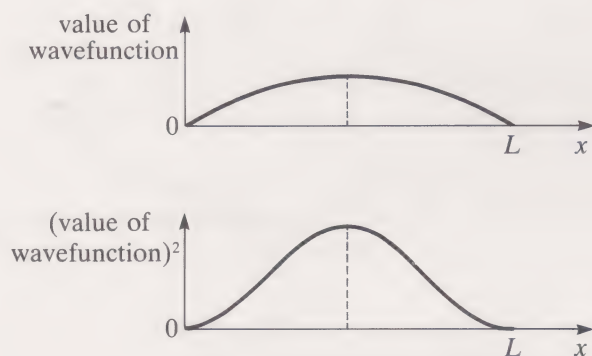
5 Examples of wavefunctions

Example 1



for every wavefunction
value outside plates = 0
so
probability of detecting the quantum
outside the plates = 0

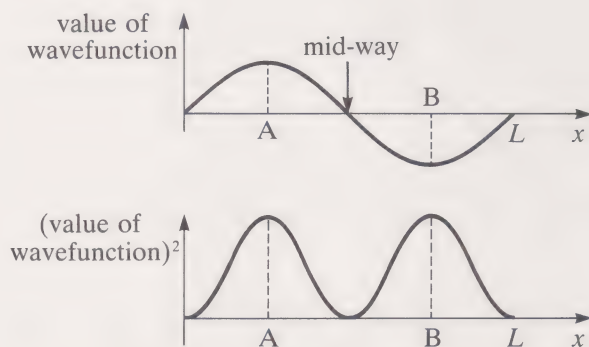
Example 2



← When the quantum is described by this wavefunction ...

... the quantum is most likely to be detected mid-way between the plates, because at this point the square of the value of wavefunction has its maximum value

Example 3



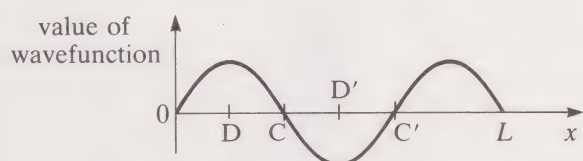
← When the quantum is described by *this* wavefunction ...

– the quantum will not be detected mid-way between the plates

– the probability of detecting the quantum at **A** = the probability of detecting it at **B**

6 Questions

Example 4



- ☐ What is the probability of detecting the quantum at **C** and at **C'**?

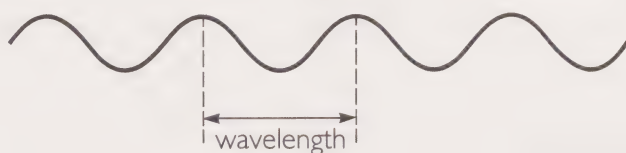
■

- ☐ Is the probability of detecting the quantum at **D** more than, less than or equal to the probability of detecting it at **D'**?

■

The probability of detecting a quantum in a small region of space depends crucially on the wavefunction that describes the quantum.

7 Infinite sine waves as wavefunctions



associated with a *free* quantum
(i.e. a quantum not subject to a net force)

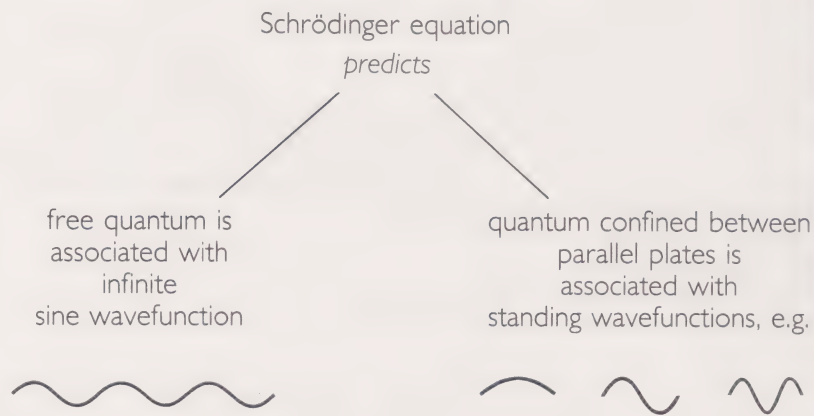
wavelength given by $\lambda_{dB} = \frac{h}{p}$

Planck's constant

magnitude of momentum

8 Calculating wavefunctions

- How can the possible wavefunctions of a quantum of matter be calculated?
- Using the Schrödinger equation.



Erwin Schrödinger
(1887–1961)

9 Summary

	waves on a rope	wavefunction
infinite sine wave		 describes a free quantum (i.e. a quantum subject to zero net force)
standing wave		 describes a quantum confined between two parallel reflecting plates

interpretation

the probability of detecting a quantum of matter in a given small region depends on the square of the value of the wavefunction in that region

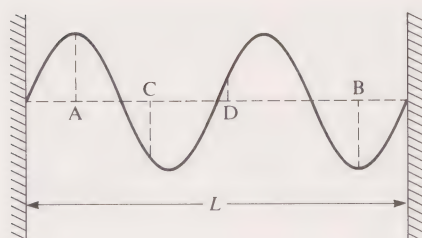


FIGURE 28 See SAQ 9.

SAQ 7 Sketch a wavefunction of (a) a proton that is subject to zero net force; (b) a neutron confined between two infinite, parallel, impenetrable reflecting plates.

SAQ 8 What is the wavelength of the wavefunction of a free electron that has a speed of $2.00 \times 10^6 \text{ m s}^{-1}$?

SAQ 9 Refer to Frame 3 of the AV sequence, which shows a quantum that is confined between two reflecting plates. If the wavefunction of the quantum is the one shown in Figure 28,

- What is the probability of detecting the quantum mid-way between the plates?
- Is the probability of detecting the quantum at A less than, greater than or equal to the probability of detecting it at B?
- Is the probability of detecting the quantum at C less than, greater than or equal to the probability of detecting it at D?

SUMMARY OF SECTION 3

- The quantum mechanical wave that is associated with a quantum of matter is called a wavefunction.
- The value of the wavefunction of a quantum of matter in a small region of space determines the probability of detecting the quantum in that region—a comparatively large magnitude of the value corresponds to a comparatively high probability of detection.
- The wavefunction of a quantum of matter that is confined between two infinite, parallel, impenetrable, reflecting plates is a standing wave.
- The wavefunction of a free quantum of matter (i.e. a quantum subject to zero net force) is an infinite sine wave.

4 HEISENBERG'S UNCERTAINTY PRINCIPLE

You saw in Section 3.1 that when a quantum is diffracted, its subsequent path cannot be predicted for certain: it is possible to predict only the *probability* that the quantum will be diffracted in a certain angular region. This uncertainty in the quantum's behaviour is a manifestation of Heisenberg's uncertainty principle, which gives fundamental limits to the accuracy with which a quantum's motion can be specified. This principle is one of the cornerstones of quantum mechanics and it cannot possibly be understood in terms of Newtonian ideas (Unit 3).

Our discussion of the principle will begin by considering in detail the motion of matter on the large scale. Then you will see how a simple mathematical form of the principle can be derived by analysing a thought experiment that concerns the diffraction of a beam of electrons by a single slit. Finally, the principle is applied to electrons in atoms.

4.1 SPECIFYING THE MOTION OF MACROSCOPIC OBJECTS

As you will see shortly, in order to discuss the Heisenberg uncertainty principle in detail, we shall need to specify carefully the motion of 'microscopic' matter, such as electrons, protons and neutrons. In order to prepare the ground for this discussion, we must now describe precisely how a descrip-

ONE-DIMENSIONAL MOTION

TWO-DIMENSIONAL MOTION

COMPONENTS OF POSITION

COMPONENTS OF VELOCITY

 COMPONENTS OF
MOMENTUM

 THREE-DIMENSIONAL
MOTION

tion of macroscopic (i.e. large-scale) objects is given in Newtonian mechanics. We shall do this by developing some of the basic ideas about mechanics that you first met in Units 3 and 9.

Consider the motion of a large-scale object of mass m along a straight line that can be labelled as the x -axis (Figure 29). This is known as **one-dimensional motion** because the position of the object can be specified at

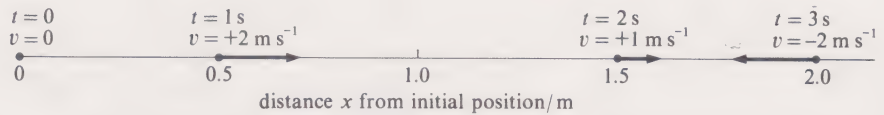


FIGURE 29 An example of one-dimensional motion. At each time, the position and velocity can each be specified by a single quantity.

each time t in terms of *one* quantity, its position x relative to its *initial* position, which is labelled $x = 0$. The velocity v of the object is represented by a bold arrow whose direction is the same as that of the object's direction of motion and whose length is proportional to the speed of the object. For the example that is illustrated in Figure 29: at $t = 1$ s (one second after the motion began), $x = 0.5$ m and $v = +2$ m s⁻¹ (to the right); at $t = 2$ s, $x = 1.5$ m and $v = +1$ m s⁻¹ (to the right); and at $t = 3$ s, $x = 2$ m and $v = -2$ m s⁻¹ (to the left). The arrow starting from the point $x = 0.5$ m is twice as long as the arrow starting from $x = 1.5$ m because the speed of the object at the former position is twice its speed at the latter position.

It is convenient to define the momentum p of an object

$$p = mv \quad (7)^*$$

You may remember from Unit 3 that momentum is important because it is conserved when a system is not subject to a net force. Moreover, we shall be particularly concerned with this variable when we discuss the uncertainty principle.

How can the concepts of position, velocity and momentum be generalized from the simple case of motion in a straight line to **two-dimensional motion**, in which the object can move *in a plane*?

Consider, for the sake of definiteness, an object of mass $m = 0.5$ kg that moves on the surface of the graph in Figure 30, having begun its motion at time $t = 0$ s, stationary at $x = 0$ m and $y = 0$ m. This may seem to be a somewhat artificial situation—motion on the surface of a piece of graph paper is hardly the most familiar of phenomena! This situation is undoubtedly artificial, but that is not important—what *is* important is that the graph and its fixed axes enable us conveniently to keep track of the object's two-dimensional motion. Look, for example, at Figures 31a and 31b, which illustrate the positions and velocities of the object at $t = 1$ s and $t = 2$ s, respectively.

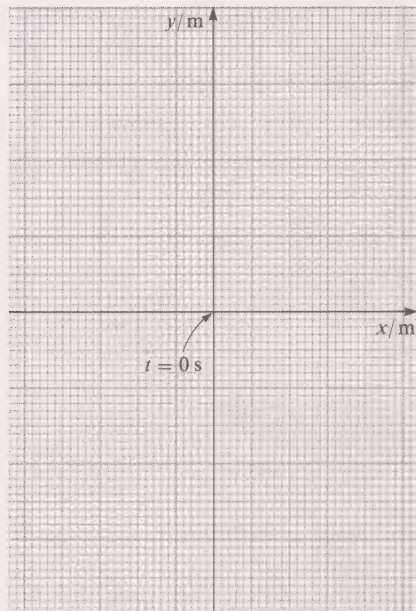


FIGURE 30 The position of a particle moving in two dimensions can be specified by x - and y -coordinates.

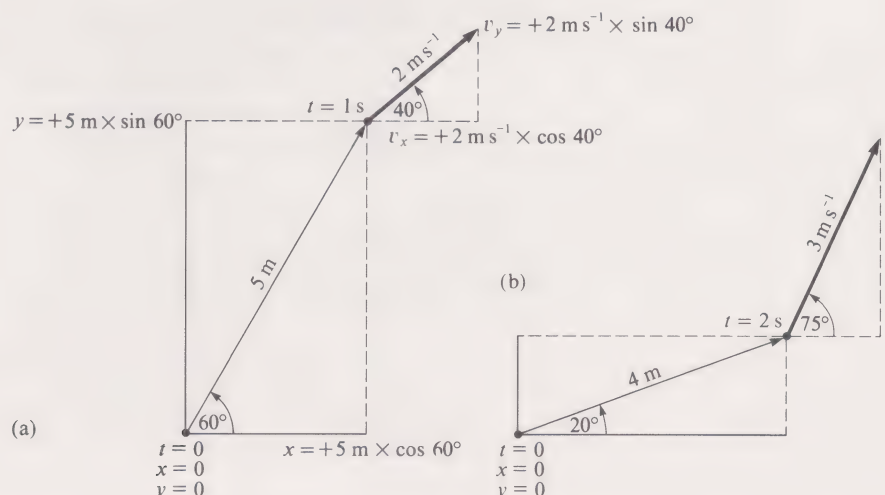


FIGURE 31 Examples of two-dimensional motion at two instants.

At $t = 1$ s (Figure 31a), the object is at a distance of 5 m from its starting point, at an angle of 60° relative to the x -axis. Its position may also be specified in terms of its values of x and y , which are known as **components of position**:

position component x = component of position in the x -direction

position component y = component of position in the y -direction

As you can see from Figure 31a, at $t = 1$ s,

$$x = 5 \text{ m} \times \cos 60^\circ = 2.50 \text{ m} \quad (9a)$$

$$y = 5 \text{ m} \times \sin 60^\circ = 4.33 \text{ m} \quad (9b)$$

What about the velocity of the object? The bold arrow on Figure 31a shows that the speed of the object (i.e. the *magnitude* of its velocity) is 2 m s^{-1} and that the velocity is directed at an angle of 40° relative to the x -axis. This is the object's direction of motion at the time—the direction in which it is moving—which should not be confused with the angle (60°) that the object position makes with the x -axis.

Just as the object's position can be specified in terms of position components (x and y), its velocity can also be specified in terms of **components of velocity**:

velocity component v_x = component of velocity in the x -direction

velocity component v_y = component of velocity in the y -direction

Figure 31a shows that, at $t = 1$ s,

$$v_x = 2 \text{ m s}^{-1} \times \cos 40^\circ = 1.53 \text{ m s}^{-1}$$

$$v_y = 2 \text{ m s}^{-1} \times \sin 40^\circ = 1.29 \text{ m s}^{-1}$$

These velocity components also enable us to calculate the **components of momentum**, p_x and p_y , of the object at $t = 1$ s:

$$p_x = mv_x = 0.5 \text{ kg} \times 1.53 \text{ m s}^{-1} = 0.77 \text{ kg m s}^{-1} \quad (10a)$$

$$p_y = mv_y = 0.5 \text{ kg} \times 1.29 \text{ m s}^{-1} = 0.65 \text{ kg m s}^{-1} \quad (10b)$$

ITQ 5 Figure 31b shows the position and velocity of the object (which has mass 0.5 kg) at $t = 2$ s. Calculate the following quantities at this time: (a) the object's position components; (b) the object's velocity components; (c) the object's momentum components.

You have now seen that one-dimensional motion (i.e. motion in a straight line) can be specified at a given time in terms of single components of position, velocity and momentum. On the other hand, two-dimensional motion (i.e. motion in a plane) can be specified in terms of pairs of components—of position (x , y), velocity (v_x , v_y) and momentum (p_x , p_y). It should not now surprise you that it is possible to specify **three-dimensional motion** (i.e. motion in *any* direction, Figure 32) in terms of triplets of components—of position (x , y , z), velocity (v_x , v_y , v_z) and momentum (p_x , p_y , p_z).

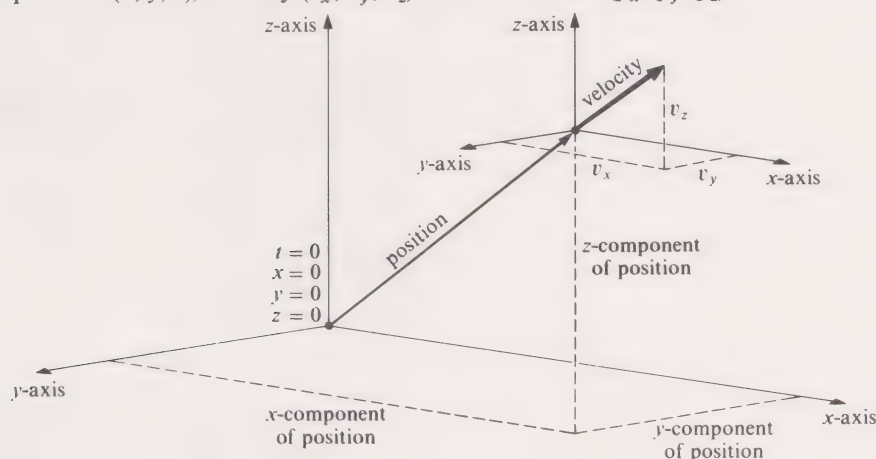


FIGURE 32 The position and velocity of a particle that executes three-dimensional motion can each be specified by three components (x , y , z) and (v_x , v_y , v_z).

UNCERTAINTY IN A
QUANTUM MECHANICAL
MEASUREMENT

So far, we have implicitly assumed that the position, velocity and momentum of the object can all be specified simultaneously (i.e. at the same time) with perfect accuracy. Our everyday experience suggests that this is a very reasonable assumption: after all, shouldn't we expect that the position and velocity of a macroscopic object, such as a tennis ball, could in principle be specified to arbitrarily high accuracy? Surely it is reasonable to say that, at any given instant, the ball is in a definite place moving with a definite velocity? We shall return to these questions (and re-examine the answers!) in Section 5, but first let's compare this 'commonsense' perception of the behaviour of macroscopic objects in our everyday world with the behaviour of a microscopic quantum, such as an electron.

 4.2 SPECIFYING THE MOTION OF A
QUANTUM—THE HEISENBERG UNCERTAINTY
PRINCIPLE

In order to introduce the Heisenberg uncertainty principle, we shall discuss a thought experiment in which a single slit diffracts a beam of free electrons (the analysis applies equally well to all other quanta). In this experiment, each electron has momentum of magnitude p , and the single slit has width w (Figure 33). Notice that the x - and y -axes on the apparatus illustrated in Figure 33 have been set up in such a way that $x = 0$ and $y = 0$ are in the centre of the slit.

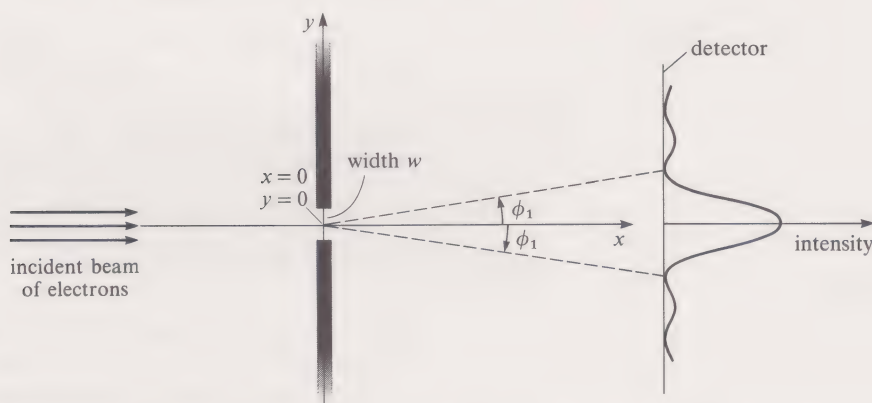


FIGURE 33 When an electron is diffracted by a single slit, it is likely (but not certain) that its angle of diffraction will be less than ϕ_1 (the angular position of the first diffraction minimum).

We shall consider measurements of the position and momentum of electrons in the beam as they pass through the slit, and for definiteness we shall concentrate on the y -components of these quantities. First, think about an experiment to measure the y -component of the positions of the electrons as they pass through the slit. Such an experiment would show that the values of y extend from $+w/2$ to $-w/2$ (Figure 34a). We say that the uncertainty in the y position component of an electron in the beam is $w/2$, and we denote this uncertainty by the symbol Δy :

$$\Delta y = \frac{w}{2} \quad (11)$$

It is important to be clear about the meaning of Δy : although this quantity gives the uncertainty in the y -component of the position of a *single* electron, Δy can be found only from a series of measurements of y taken with *many* electrons as they pass through the slit.

We can now generalize the concept of quantum mechanical uncertainty. For a given experimental set-up, the **uncertainty in a quantum mechanical measurement** of a quantity may be taken to be half the spread of the results of the measurements of the quantity. More rigorously, the uncertainty is the standard deviation of the measurements (Unit 4).

We are now in a position to derive a form of the Heisenberg uncertainty principle. First, we shall find an expression for the uncertainty Δp_y in the momentum component p_y of an electron as it passes through the slit, and then we shall multiply this expression by the uncertainty Δy (Equation 11).

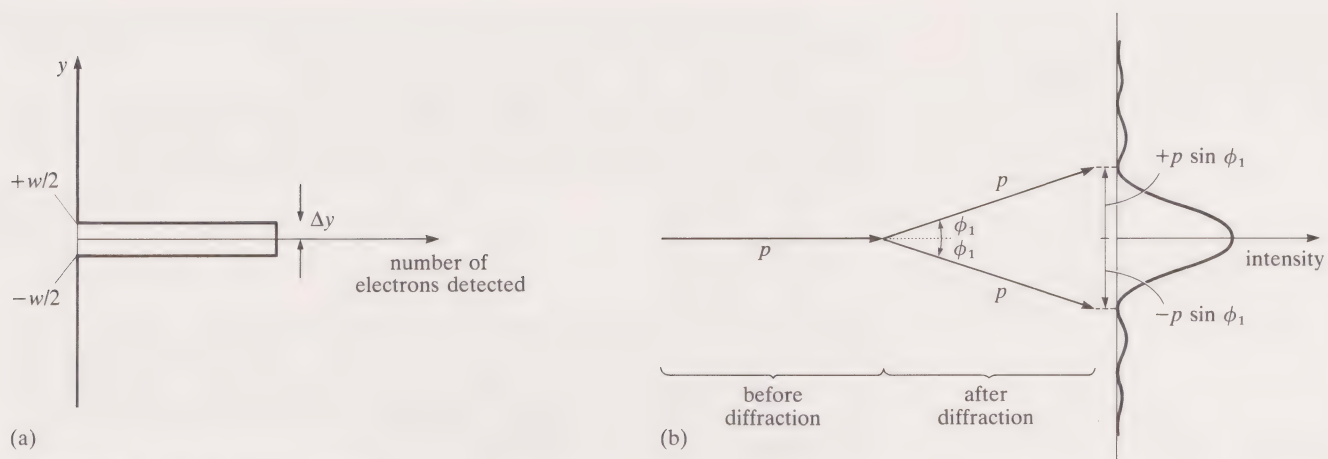


FIGURE 34 (a) If, for a number of electrons, the y -component of position is measured, as the electrons pass through a slit of width w , the results will be evenly spread from $y = +w/2$ to $-w/2$. The uncertainty Δy is $w/2$, half the spread w . (b) The y -component p_y of the momentum of the electrons is *likely* to lie between $+p \sin \phi_1$ and $-p \sin \phi_1$ (some values of p_y will lie *outside* this range because the diffraction pattern extends past the diffraction angle ϕ_1).

As you will see shortly, this particular combination of uncertainties gives a very simple result.

In order to find the uncertainty Δp_y in the y -component of the momentum of an electron in the beam, we need to consider where the electrons strike the screen after they have passed through the slit.

□ At what angle will an electron in the beam be diffracted?

■ It is impossible to say. As you saw in Section 3, it is possible to give only the *probabilities* that the electron will arrive in different small regions of the screen.

Although it is not possible to predict the angle at which an electron in the beam is diffracted, it *is* possible to say that there is a high probability that the electron will be detected somewhere in the central diffraction peak, i.e. that the diffraction angle is likely to be between 0 and ϕ_1 , either side of the straight-through position (Figure 33). This implies that measurements of the y -component p_y of momentum *after* diffraction will *probably* be between $+p \sin \phi_1$ and $-p \sin \phi_1$ (Figure 34b). Hence, it is reasonable to assert that the uncertainty Δp_y is approximately $p \sin \phi_1$ (i.e. half the spread):

$$\Delta p_y \approx p \sin \phi_1 \quad (12)$$

The quantity $\sin \phi_1$ is given by

$$\sin \phi_1 = \frac{\lambda_{dB}}{w} \quad (13)$$

where λ_{dB} is the de Broglie wavelength of the free electron. Equation 13 is precisely analogous to the corresponding expression for the diffraction of light by a single slit, which you met in Unit 10. When the expression for $\sin \phi_1$ in Equation 13 is substituted into Equation 12 we find:

$$\Delta p_y \approx p \frac{\lambda_{dB}}{w} \quad (14)$$

In Section 2, the de Broglie wavelength of a free quantum was given as

$$\lambda_{dB} = \frac{h}{p} \quad (6)^*$$

When this expression is substituted into Equation 14, we find

$$\Delta p_y \approx \frac{h}{w} \quad (15)$$

The product of Δy and Δp_y is therefore given by

$$\Delta y \times \Delta p_y \approx \frac{w}{2} \times \frac{h}{w} \quad (16)$$

using Equations 11 and 15. Hence,

$$\Delta y \Delta p_y \approx \frac{h}{2} \quad (17)$$

HEISENBERG'S
UNCERTAINTY PRINCIPLE

So, according to our analysis, the product of the uncertainties Δy and Δp_y is approximately $h/2$, half the value of Planck's constant.

By using a more rigorous analysis, it can be shown that the product $\Delta y \Delta p_y$ is actually *greater than or equal to* Planck's constant h divided by 4π . In mathematical notation, the correct relationship between Δy and Δp_y is

$$\Delta y \Delta p_y \geq \frac{h}{4\pi} \quad (18a)$$

where the sign $\geq$ means 'greater than or equal to', that is, 'at least'. This relationship is quite general—it applies to *all* quanta in *all* circumstances, not only to electrons that are diffracted by a single slit. Also, the relationship applies equally well to the other two components of position and momentum:

$$\Delta x \Delta p_x \geq \frac{h}{4\pi} \quad (18b)$$

$$\Delta z \Delta p_z \geq \frac{h}{4\pi} \quad (18c)$$

Notice that the principle refers to the products of position components and the *corresponding* momentum components—it says nothing about products such as $\Delta x \Delta p_y$, $\Delta z \Delta p_y$ and $\Delta y \Delta p_z$. The relationships in Equations 18a–c are known as *Heisenberg's uncertainty relations* after Werner Heisenberg (Figure 35), who was the first to formulate them, in 1927. The relations embody **Heisenberg's uncertainty principle**, or 'the uncertainty principle' for short.

HEISENBERG'S UNCERTAINTY PRINCIPLE

The product of the uncertainty in a component of the position of a quantum (e.g. Δy) and the uncertainty in the corresponding component of the quantum's momentum at the same time (e.g. Δp_y) is at least $h/4\pi$, e.g.

$$\Delta y \Delta p_y \geq \frac{h}{4\pi} \quad (18a)^*$$

The principle also applies to certain other pairs of variables (for example, energy and time) but we shall not be concerned with these other forms of the principle in this Course.

At first sight, the principle may appear only to give technical limitations on the accuracy with which the motion of a quantum can be specified. However, the principle is more important than that—it is of fundamental significance in quantum physics. And, as the following example shows, the principle has some truly remarkable consequences.

Consider a quantum, say a proton, whose x -component of position is known with perfect accuracy. What does the uncertainty principle have to say about the x -component of momentum of the proton *at the same time*? Well, since the x -component of the proton's position is assumed to be known with perfect accuracy, it follows that the uncertainty Δx in the proton's x -component of position is zero

$$\Delta x = 0 \quad (19)$$

The appropriate uncertainty relation, Equation 18b, then implies that

$$\Delta p_x \geq \frac{h}{4\pi \Delta x} \quad (20)$$

$$\text{so } \Delta p_x = \infty \quad (21)$$

because $h/0 = \infty$, infinity. Hence, there is in this case infinite uncertainty in p_x , which implies that nothing at all is known about the value of p_x . This remarkable result is worth summarizing—if the x -component of position of

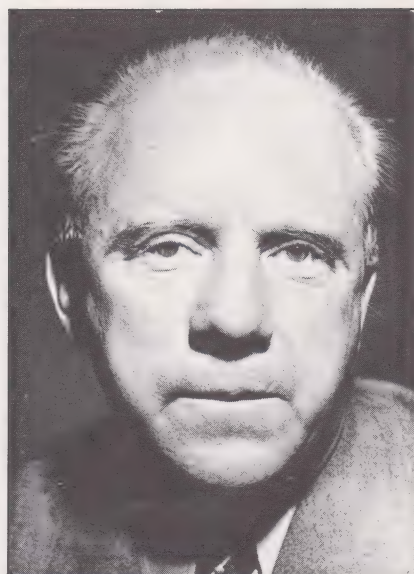


FIGURE 35 Werner Heisenberg (1901–1976) was awarded the Nobel Prize for physics in 1932 for 'his discovery of the uncertainty principle'. Although he was undoubtedly a great theoretician, it appears that his practical skills left something to be desired. In his PhD oral examination, his responses to questions on experimental physics were so poor that one of his examiners, the Nobel Prize winning physicist Wilhelm Wien, recommended that he should fail! (He was eventually awarded a pass for the exam—grade three.)

a quantum is known with perfect accuracy ($\Delta x = 0$), it follows from the uncertainty principle that nothing at all can be said for certain about the x -component of the quantum's momentum at the same time. (Similar reasoning can be applied to the other pairs of components: e.g. if $\Delta z = 0$, then $\Delta p_z = \infty$.) Hence, if you know exactly where a quantum is, you cannot know both how quickly, and in which direction, it is moving!

ITQ 6 Consider a quantum, say an electron, whose y -component of momentum is known with perfect accuracy. What does the Heisenberg uncertainty principle say about the accuracy with which the y -component of the quantum's position is known at the same time?

You have now seen that the uncertainty principle implies that the accuracy with which a position component of a quantum can be specified depends crucially on the accuracy with which the corresponding momentum component is specified (and vice versa). If one of the quantities is known very accurately, the other must perforce be known very *inaccurately*.

You may well be wondering if this restriction on the accuracies with which position and momentum components can be specified can be overcome by sufficiently ingenious experimental design. In fact, it cannot—the restriction is *fundamental* in the sense that it reflects the behaviour of nature, not a deficiency in the design of measuring apparatus. According to the uncertainty principle, no matter how well the apparatus is designed and constructed, the restriction on the accuracies with which position components and corresponding momentum components can be specified simultaneously *cannot be overcome*.

If you are one of those who find this difficult to accept, don't worry, you have been in good company. Einstein always believed that it should be possible *in principle* to measure position components, momentum components and all other variables of every quantum simultaneously to arbitrarily high accuracy, given sufficiently accurate and well-designed equipment. Heisenberg has described how, at a conference in 1927, Einstein went to considerable trouble to convince his friend and colleague Niels Bohr (one of the great pioneers of quantum physics) that the uncertainty principle is not valid.

And so he [Einstein] refused point-blank to accept the uncertainty principle, and tried to think up cases in which the principle would not hold.

The discussion usually started at breakfast, with Einstein serving us up with yet another imaginary experiment by which he thought he had definitely refuted the uncertainty principle. We would at once examine his fresh offering, and on the way to the conference hall, to which I generally accompanied Bohr and Einstein, we would clarify some of the points and discuss their relevance. Then, in the course of the day, we would have further discussions on the matter, and, as a rule, by suppertime we would have reached the point where Niels Bohr could prove to Einstein that even his latest experiment failed to shake the uncertainty principle.

(Heisenberg, 1949)



Einstein never did manage to refute the uncertainty principle. Indeed, no one has ever been able to show that it is theoretically or experimentally invalid—Heisenberg's uncertainty principle is now universally accepted and is one of the tools of the trade for the professional quantum mechanic.

4.3 APPLYING THE UNCERTAINTY PRINCIPLE TO ATOMS

You saw in Units 11–12 that each atom consists of electrons moving around a charged nucleus (except the hydrogen atom, which contains only one electron). According to commonsense ideas, electrons in a many-electron atom can each be visualized as having a clearly defined path

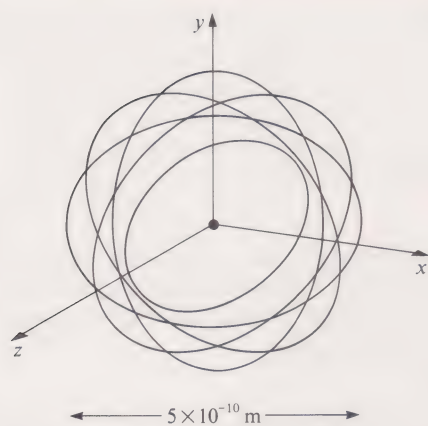


FIGURE 36 A simple visualization of an atom, according to which each electron has a clearly defined path (A set of x -, y - and z -axes has been set up with the origin at the centre of the nucleus.)

(Figure 36). In such a visualization, it is implicitly assumed that the position and direction of motion of each electron are known with perfect accuracy at the instant at which the illustration applies. However, the uncertainty principle says that such an accurate specification is not possible.

So how accurately *can* the position and momentum of an electron be simultaneously specified? In order to answer this question, we shall consider the uncertainties that would arise in experiments to measure, at a given instant, the position and momentum of an electron in a typical atom, which has a diameter of $5 \times 10^{-10} \text{ m}$. For convenience, we shall draw x -, y - and z -axes in the atom that we are considering, with $x = y = z = 0$ at the centre of the nucleus.

We shall now use the uncertainty principle to show that the confinement of an electron in an atom implies that there is a considerable uncertainty in each component of the electron's velocity. The discussion will concentrate on motion relative to the x -axis, although the arguments apply equally well to motion with respect to the other two axes.

The uncertainty Δx in the x -component of the position of an electron in an atom may reasonably be estimated as being $2.5 \times 10^{-10} \text{ m}$, approximately equal to half the typical diameter of an atom (Figure 36):

$$\Delta x = 2.5 \times 10^{-10} \text{ m} \quad (22)$$

Now that we have an estimate for Δx , the uncertainty principle allows us to find Δp_x , the uncertainty in the corresponding specification of the x -component of the momentum of the electron. Because $\Delta x \Delta p_x \geq h/4\pi$ (Equation 18b) it follows that

$$\Delta p_x \geq \frac{h}{4\pi \Delta x} \quad (20)^*$$

$$\text{i.e. } \Delta p_x \geq \frac{6.63 \times 10^{-34} \text{ J s}}{4 \times 3.14 \times 2.50 \times 10^{-10} \text{ m}} \quad (23)$$

$$\text{so}^\dagger \Delta p_x \geq 2.11 \times 10^{-25} \text{ kg m s}^{-1} \quad (24)$$

The uncertainty Δp_x in the x -component p_x of the momentum of the electron is related to the uncertainty Δv_x in the x -component v_x of the electron's velocity using Equation 7:

$$\Delta p_x = m_e \Delta v_x \quad (25)$$

Note that we are assuming that the *uncertainty* Δm_e in the mass of the electron is negligible.

From Equation 25, the uncertainty Δv_x in the x -component of the electron's velocity is

$$\Delta v_x = \frac{\Delta p_x}{m_e} \quad (26)$$

so, using this equation and the result in Equation 24,

$$\Delta v_x \geq \frac{2.11 \times 10^{-25} \text{ kg m s}^{-1}}{9.11 \times 10^{-31} \text{ kg}} \quad (27)$$

$$\text{i.e. } \Delta v_x \geq 2.3 \times 10^5 \text{ m s}^{-1} \quad (28)$$

This is a remarkable conclusion: the uncertainty Δv_x in the x -component of velocity of an electron in an atom is at least 230 thousand metres per second! This result has followed directly from the Heisenberg uncertainty principle in conjunction with the reasonable assumption that $\Delta x = 2.5 \times 10^{-10} \text{ m}$ for an electron in an atom. Because Δy and Δz are also

[†] The standard SI unit of momentum, kg m s^{-1} , is equivalent to the unit J s m^{-1} : you saw in Unit 9 that $1 \text{ J} = 1 \text{ N m} = 1 \text{ kg m}^2 \text{ s}^{-2}$, so $1 \text{ J s m}^{-1} = 1 \text{ kg m s}^{-1}$.

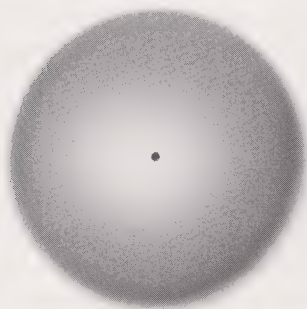


FIGURE 37 A quantum mechanical visualization of an atom (the electrons are visualized as a fuzzy cloud).

approximately equal to 2.5×10^{-10} m, it follows that Δv_y and Δv_z are greater than or approximately equal to 230 thousand metres per second as well.

The uncertainty principle therefore says that because the electron in an atom is confined to a tiny region of space, it follows that the corresponding *uncertainty in each component of the electron's velocity is more than two hundred thousand metres per second!* This is in sharp contrast with expectations based on Newtonian mechanics, according to which the electron's velocity could in principle be known with *zero* uncertainty! It should now be plain to you that the picture of the atom given in Figure 36 must be radically revised. Somehow, it must be shown that the speed and direction of motion of each confined electron is very uncertain.

The conventional quantum mechanical visualization of an atom is shown in Figure 37. The electrons are shown as a cloud whose fuzziness symbolizes the limits on our knowledge of the position and momentum of each electron at a given time (the same picture is used to visualize a hydrogen atom). Aside, it is worth remarking that the picture of the atom in Figure 37 is consistent with what you learned about quanta of matter in the AV sequence 'Wavefunctions of matter'. An atomic electron is described by a wavefunction that, as you will see in Unit 31, spreads across the whole atom, and this wavefunction gives the probability of detecting the electron in each tiny region of the atom.

The Heisenberg uncertainty principle has one other very important implication—it implies that there is no state of an atom in which a constituent electron will be observed to be permanently at rest. In order to understand why, suppose for a moment that an atomic electron *did* permanently have zero velocity. This would imply that the uncertainty in the electron's momentum components were each zero ($\Delta p_x = \Delta p_y = \Delta p_z = 0$) so, according to the uncertainty principle, there would be *infinite* uncertainty in the electron's position components, $\Delta x = \Delta y = \Delta z = \infty$ (ITQ 6)! This cannot be the case because atomic electrons are normally confined to a *tiny* region of space ($\Delta x = \Delta y = \Delta z = 2.5 \times 10^{-10}$ m). Hence, the initial suggestion—that there exist atomic states in which a constituent electron permanently has zero velocity—cannot be true. For any atomic state, measurements of the electron's velocity will give a range of values with a spread consistent with the uncertainty principle. This is a purely quantum effect, and it cannot possibly be understood within the framework of Newtonian mechanics.

SUMMARY OF SECTION 4

1 According to ideas based on Newtonian mechanics, it is possible to give an accurate description of the motion of a particle (at each instant) by specifying the position and momentum of the particle to arbitrarily high accuracy.

2 According to the Heisenberg uncertainty principle, one of the cornerstones of quantum mechanics, if there is an uncertainty Δx in the instantaneous x-component of the position of a quantum, then the uncertainty Δp_x in the corresponding x-component of the quantum's momentum is constrained by the requirement

$$\Delta x \Delta p_x \geq \frac{h}{4\pi} \quad (18b)^*$$

and the same is true for the y- and z-components of position and momentum. These constraints on our knowledge of position and momentum components cannot be overcome by using better experimental apparatus to measure the quantities.

3 The uncertainty principle implies that the electrons in an atom (or the single electron in a hydrogen atom) should be visualized as a fuzzy cloud that symbolizes the uncertainties in the position and momentum of each electron.

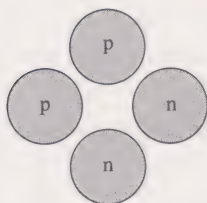


FIGURE 38 A simplistic visualization of a helium nucleus.

SAQ 10 At a party in Islington, you meet an amateur philosopher who tells you that ‘according to the Heisenberg uncertainty principle, everything in physics is uncertain’. You are dismayed by the superficiality of this statement and you resolve to correct it. In simple, non-mathematical terms, explain to your fellow guest the true meaning of the uncertainty principle.

SAQ 11 In an elementary textbook on nuclear physics, a helium nucleus (which contains two protons and two neutrons and has a diameter of approximately 10^{-14} m) is visualized as in Figure 38.

(a) What is the uncertainty in each component of the velocity of one of the quanta (a proton or neutron) in the nucleus? (Use the data on the back cover and calculate your answer to two significant figures.)

(b) Use your answer to part (a) to suggest a better way of visualizing the helium nucleus.

SAQ 12 Using the Heisenberg uncertainty principle, explain why there is no state of a nucleus in which a constituent proton or neutron is permanently at rest.

SAQ 13 You saw in the AV sequence that the magnitude of the momentum of a free quantum has a definite value p and that the wavelength of the quantum’s wavefunction is given by $\lambda_{dB} = h/p$. In terms of the uncertainty principle, explain qualitatively why the wavefunction of the quantum must be of infinite extent.

5 QUANTUM MECHANICS AND THE LARGE-SCALE WORLD

At the end of Section 4.1, we pointed out that in the everyday world we *assume* that we can specify the position and velocity of objects (such as tennis balls) to an accuracy limited only by our measuring techniques and our equipment. But is the assumption correct, or does quantum mechanics apply to macroscopic objects as well as to microscopic quanta? In this short Section, the answer to this question will be given and one of its consequences will be discussed in detail.

5.1 DOES QUANTUM MECHANICS APPLY TO MACROSCOPIC OBJECTS?

The short answer to this question is Yes: *quantum mechanics is believed to apply to all matter**. This has some profound implications for our understanding of the everyday world.

As you have seen, according to quantum mechanics

- (a) each quantum of matter is associated with a wavefunction;
- (b) the accuracies with which the quantum’s position and momentum components can be simultaneously specified are limited by the Heisenberg uncertainty principle;
- (c) the quantum can be diffracted.

Because quantum mechanics is believed to apply equally well to macroscopic objects as to microscopic quanta (electrons, protons, atoms, etc.), it follows that *the above results should also apply to macroscopic objects*.

* With the proviso that the matter is not moving with a speed close to that of light in a vacuum, in which case a *relativistic* version of quantum mechanics must be used.



FIGURE 39 Someone walking through a doorway.

You may well be very sceptical of that last statement! Does the uncertainty principle *really* apply in the everyday world? Can a macroscopic object be diffracted? In Section 5.2, the latter question will be considered in detail.

5.2 DIFFRACTION OF MACROSCOPIC OBJECTS

The sight of someone walking through a doorway (Figure 39) is common enough. Nothing appears to happen to the walker on passing through the doorway, and their path is independent of the doorway's width (provided that it is wide enough for the walker to pass through without coming into contact with the door frame). However, quantum mechanics leads us to expect that this situation is not as simple as it first appears to be. The person walking through the doorway has an associated wave and the doorway itself may be regarded as a single slit (Figure 40). Hence, the walker should be diffracted! Yet if this is so, why haven't you ever been conscious of being diffracted when you pass through a doorway?

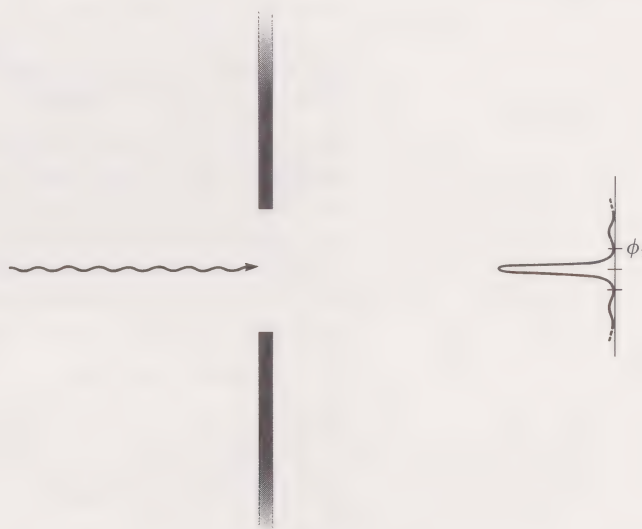


FIGURE 40 The situation shown in Figure 39 can be modelled quantum mechanically as a wave passing through a single slit.

Let us examine the situation more closely. Suppose that the walker is someone with a mass of 50 kg, walking with a speed of 1 m s^{-1} through a doorway of width 0.80 m. In order to estimate the maximum probable angle of diffraction, we shall assume that the walker can be regarded as a free particle. The de Broglie formula can then be used to calculate their wavelength.

ITQ 7 What is the walker's de Broglie wavelength λ_{dB} ?

You saw in ITQ 7 that the walker's de Broglie wavelength λ_{dB} is very much smaller than the width of the doorway. Hence, you should expect immediately that they will not be significantly diffracted; diffraction will be significant only if the wavelength is not much smaller than the width of the slit.

- ☐ Is it possible to predict precisely the angle at which the walker will be diffracted?
- ☒ No, but it is possible to predict the probabilities with which the walker will be diffracted in a given angular region. Also, it is likely that the walker will be diffracted by an angle that is less than ϕ_1 , the angle at which the first diffraction minimum occurs (Figure 40). This macroscopic situation is exactly analogous to the microscopic example illustrated in Figure 33.

The angle ϕ_1 may be estimated using Equation 13

$$\sin \phi_1 = \frac{\lambda_{dB}}{w} \quad (13)^*$$

where w is the width of the doorway. Because $\lambda_{dB} = 1.3 \times 10^{-35} \text{ m}$ (ITQ 7) and $w = 0.80 \text{ m}$, Equation 13 implies that

$$\sin \phi_1 = \frac{1.3 \times 10^{-35} \text{ m}}{0.80 \text{ m}}$$

$$\sin \phi_1 = 1.6 \times 10^{-35}$$

$$\text{i.e. } \phi_1 \sim 10^{-33} \text{ degree}$$

Hence, the walker would be diffracted by an angle of no more than about 10^{-33} degree, i.e. a thousand-million-million-million-million-millionth of a degree! This is such a small angle that the diffraction could not possibly be observed.

This simple calculation illustrates an important general result: *quantum mechanics predicts that although macroscopic objects are diffracted, the angles at which such objects are diffracted are negligible*. The diffraction of macroscopic objects can generally be neglected because of the smallness of their de Broglie wavelengths.

Just as the diffraction of matter is observable on the microscopic scale but negligible on the large scale, so most other quantum mechanical results that are crucial in allowing an understanding of electrons, protons, atoms, etc., are unobservable in everyday life. For example, the Heisenberg uncertainty principle can be applied to macroscopic objects but the results do not give significant practical limitations to the accuracies with which the objects' positions and momenta may be specified.

It is generally true that in the everyday world the predictions of quantum mechanics agree almost perfectly with the corresponding predictions of Newtonian mechanics, which describes extremely well the motion of macroscopic objects, as you saw in Unit 3. There are, however, many macroscopically observed phenomena that can be explained only by applying quantum mechanics to the microscopic constituents of matter. A classic example of this is the phenomenon of superconductivity, in which the resistance to electric current of some substances falls to zero below a critical temperature (each of these substances has a characteristic value of this temperature). Superconductivity was discovered in 1911 by the Dutch physicist Kamerlingh Onnes, but it was not until 1957 that the phenomenon was explained quantum mechanically by the American physicists John Bardeen, Leon Cooper and John Schrieffer.

You should leave this Section in no doubt that quantum mechanics is superior to Newtonian mechanics as a theory of matter. Whereas Newton's theory can be relied upon to account successfully for the motion only of large-scale objects, quantum mechanics describes the behaviour of all matter—microscopic and macroscopic.

SUMMARY OF SECTION 5

- 1 Quantum mechanics applies to all matter, microscopic quanta (e.g. electrons, atoms and molecules) and macroscopic objects. Hence, quantum mechanics is superior to Newtonian mechanics, which accounts successfully only for the motion of macroscopic objects.
- 2 Quantum mechanics predicts that macroscopic objects can be diffracted, but diffraction effects for such objects are far too small to be observable.
- 3 The motion of macroscopic objects can be predicted using quantum mechanics, but these predictions differ negligibly from those of Newtonian mechanics.

SAQ 14 Which of the following situations do you expect to be successfully accounted for (i) by quantum mechanics and (ii) by Newtonian mechanics?

- (a) The motion of a neutron in a uranium nucleus;
- (b) The behaviour of a carbon atom in a carbon dioxide molecule;
- (c) The motion of a football across Wembley stadium;
- (d) An apple falling from a tree.

SAQ 15 At the beginning of this text, we said that the statements below are strictly speaking false, although they appear (according to common sense) to be true. Explain briefly why each statement is wrong according to quantum mechanics.

- 1 When someone walks through a doorway without touching the door frame, the width of the doorway does not affect the walker's subsequent path.
- 2 When a stone is dropped down a well, it is possible to predict for certain where the stone will strike the bottom of the well.

6 EPILOGUE TO UNIT 30

In this Unit, you have seen how quantum ideas have revolutionized our view of matter. We began with a revision of the wave and particle models of electromagnetic radiation. Both models are incorporated in quantum electrodynamics. The propagation and interactions of *matter* can also be understood fully only if both wave *and* particle models are used. Quantum mechanics allows the propagation and interactions of matter to be understood within the framework of a single theory, a theory of the *behaviour* of matter. Quantum mechanics does not deal with a sample of matter simply as a particle or as a wave, rather it deals with matter as a particle when it interacts and as a wave when it propagates.

We have not shown how the theory of quantum mechanics is formulated—that would have been beyond the scope and level of this Course—but we have discussed some of the theory's principal results and we have described how they are interpreted.

The quantum mechanical ideas that you have met were proposed in the 1920s, nearly 250 years after Newton formulated his theory of mechanics. Newton could not have developed quantum mechanics because neither he nor his contemporaries had apparatus that could motivate the need for such a theory or check its predictions. However, that is not to belittle Newton's work, which was triumphantly successful by any standards. It gave many profound insights to the scientists who developed quantum ideas—clearly, if quantum mechanics was to be a worthy successor to Newtonian mechanics, it was essential for the new theory to reproduce the success of its predecessor in accounting for the behaviour of macroscopic objects. In Section 5, you saw that quantum mechanics was indeed successful in this way—it could be applied to matter in the large-scale world, although quantum effects are far too small to be observed for macroscopic objects.

This implied that a number of conventional ideas about the behaviour of matter must be abandoned. For example, quantum mechanics says that

When you walk through a doorway, you are diffracted;

When you are diffracted, your angle of diffraction cannot be predicted exactly, even in principle;

If you know exactly where you are, you cannot possibly know both your speed and the direction in which you are moving.

Do you still trust your common sense?

OBJECTIVES FOR UNIT 30

After you have worked through this Unit, you should be able to

- 1 Explain the meaning of, and use correctly, all the terms flagged in the text.
- 2 Recall evidence for the wave and particle models of matter. (SAQs 1 and 4)
- 3 Recall the de Broglie formula $\lambda_{dB} = h/p$ and apply it to simple problems of physics. (ITQ 3 and 7; SAQs 2 and 8)
- 4 Recall the range of applicability of quantum mechanics. (SAQs 5, 14 and 15)
- 5 Interpret the diffraction pattern observed when matter is diffracted. (ITQ 4 and SAQ 6)
- 6 Sketch the wavefunctions of matter that is (a) free, (b) confined between parallel plates; also, interpret the wavefunctions in the latter case. (AV; SAQs 7 and 9)
- 7 Calculate the position, velocity and momentum components of a macroscopic object that moves in one or two dimensions (ITQ 5)
- 8 State the Heisenberg uncertainty principle and apply it to simple physical situations, e.g. electrons in atoms, protons and neutrons in nuclei. (ITQ 6, SAQs 10–13)
- 9 Apply simple quantum mechanical ideas to macroscopic objects (ITQ 7, SAQs 14 and 15).

ITQ ANSWERS AND COMMENTS

ITQ 1 (a) Diffraction effects will be more clearly evident in the experiment with the grating whose spacing is 1400 nm: diffraction effects are significant only if the grating spacing is not much larger than the wavelength of the incident radiation. (Note that 1 mm, i.e. 10^{-3} m, is much greater than 700 nm, 7×10^{-7} m.)

(b) For the grating with a spacing of 1400 nm, $\theta_1 = 30^\circ$; for the grating with a spacing of 10^{-3} m, $\theta_1 = 0.040^\circ$.

Using Equation 1, $n\lambda = d \sin \theta_n$, with $n = 1$

$$\sin \theta_1 = \frac{\lambda}{d}$$

Hence, when $\lambda = 700$ nm and $d = 1400$ nm

$$\sin \theta_1 = \frac{700 \text{ nm}}{1400 \text{ nm}} = 0.5, \quad \text{i.e. } \theta_1 = 30^\circ$$

and when $d = 1$ mm,

$$\sin \theta_1 = \frac{7.00 \times 10^{-7} \text{ m}}{1.00 \times 10^{-3} \text{ m}} = 7.00 \times 10^{-4},$$

i.e. $\theta_1 = 0.040^\circ$.

This is consistent with the answer to part (a): diffraction effects are more clearly evident (i.e. θ_1 is greater) in the experiment with the grating whose spacing is 1400 nm.

ITQ 2 The energy of each photon in the beam is 2.8×10^{-19} J. Using Equation 4, the energy E of each

photon is

$$E = \frac{(6.63 \times 10^{-34} \text{ J s}) \times (3.00 \times 10^8 \text{ m s}^{-1})}{7.00 \times 10^{-7} \text{ m}}$$

i.e. $E = 2.8 \times 10^{-19}$ J

ITQ 3 Using Equation 5 ($p = E/c$ for a photon),

$$\frac{h}{p} = \frac{hc}{E}$$

Because $E = hf$ for a photon (Equation 3), it follows that

$$\frac{hc}{E} = \frac{hc}{hf} = \frac{c}{f}$$

Equation 2 tells us that $c/f = \lambda$, so h/p (the quantity with which we began) is equal to the wavelength λ of the radiation. In other words, for electromagnetic radiation, the de Broglie wavelength λ_{dB} is equal to the ordinary wavelength λ of the radiation.

ITQ 4 About 3.7 times more likely, because the intensity in the straight-through position is approximately 3.7 times the intensity at a diffraction angle of 2° . The intensity in the straight-through position is approximately 22 units, and the intensity when the diffraction angle is 2° is approximately 6 units. The ratio of the intensities is, therefore, $22/6 \approx 3.7$.

ITQ 5 (a) $x = 3.76 \text{ m}$, $y = 1.37 \text{ m}$:
position component $x = 4 \text{ m} \times \cos 20^\circ = 3.76 \text{ m}$
position component $y = 4 \text{ m} \times \sin 20^\circ = 1.37 \text{ m}$

(b) $v_x = 0.78 \text{ m s}^{-1}$, $v_y = 2.90 \text{ m s}^{-1}$;
velocity component $v_x = 3 \text{ m s}^{-1} \times \cos 75^\circ = 0.78 \text{ m s}^{-1}$
velocity component $v_y = 3 \text{ m s}^{-1} \times \sin 75^\circ = 2.90 \text{ m s}^{-1}$

Notice that the direction of motion of the object at $t = 2 \text{ s}$ is *not* the same as its direction of motion at $t = 1 \text{ s}$.

(c) $p_x \approx 0.39 \text{ kg m s}^{-1}$, $p_y \approx 1.45 \text{ kg m s}^{-1}$;
momentum component $p_x = mv_x = 0.5 \text{ kg} \times 0.78 \text{ m s}^{-1}$
 $\approx 0.39 \text{ kg m s}^{-1}$
momentum component $p_y = mv_y = 0.5 \text{ kg} \times 2.90 \text{ m s}^{-1}$
 $\approx 1.45 \text{ kg m s}^{-1}$

using the answers to part (b).

ITQ 6 Nothing can be known about y at the time at which p_y is known for certain. Because, in this case, $\Delta p_y = 0$, the Heisenberg uncertainty relation $\Delta y \Delta p_y \geq h/4\pi$ (Equation 18a) implies that

$$\Delta y \geq \frac{h}{0} = \infty$$

Hence, y must be infinitely *uncertain* when p_y is known for certain.

ITQ 7 $\lambda_{\text{dB}} = 1.3 \times 10^{-35} \text{ m}$

Using Equation 8, $\lambda_{\text{dB}} = h/mv$ (m denotes mass, v denotes speed and h is Planck's constant)

$$\lambda_{\text{dB}} = \frac{6.63 \times 10^{-34} \text{ J s}}{(50.0 \text{ kg}) \times (1.00 \text{ m s}^{-1})}$$

i.e. $\lambda_{\text{dB}} = 1.3 \times 10^{-35} \text{ m}$

SAQ ANSWERS AND COMMENTS

SAQ 1 The photoelectric effect is an *absorption* process: all of the incident photon's energy is transferred to the electron, and the photon 'disappears'. The Compton effect is a *scattering* process; there are two particles present both before and after the collision. The Compton effect can be understood by applying the laws of conservation of energy and momentum to the process.

SAQ 2 (a) 21° .

Using Equation 1 with $n = 1$, $\sin \theta_1 = \lambda/d$, i.e.

$$\sin \theta_1 = \frac{5.89 \times 10^{-7} \text{ m}}{1.67 \times 10^{-6} \text{ m}} = 0.353$$

i.e. $\theta_1 = 21^\circ$ (to two significant figures).

(b) 0.025° .

The de Broglie wavelength λ_{dB} of the free electrons is given by Equation 8

$$\lambda_{\text{dB}} = \frac{h}{mv}$$

where h is Planck's constant and where m and v respectively denote mass and speed. For electrons with a speed of $1.00 \times 10^6 \text{ m s}^{-1}$,

$$\lambda_{\text{dB}} = \frac{6.63 \times 10^{-34} \text{ J s}}{(9.11 \times 10^{-31} \text{ kg}) \times (1.00 \times 10^6 \text{ m s}^{-1})}$$

$$\lambda_{\text{dB}} = 7.28 \times 10^{-10} \text{ m}$$

The angle θ_1 at which the first diffraction maximum would be observed is given by $\sin \theta_1 = \lambda_{\text{dB}}/d$ (the wavelength λ of the radiation in the corresponding equation in part (a) has been replaced by the de Broglie wavelength λ_{dB} of the free electrons). In this case,

$$\sin \theta_1 = \frac{7.28 \times 10^{-10} \text{ m}}{1.67 \times 10^{-6} \text{ m}} = 4.36 \times 10^{-4}$$

i.e. $\theta_1 = 0.025^\circ$ (to two significant figures).

Note that θ_1 for the beam of free electrons would be very much smaller than θ_1 for the beam of sodium light.

SAQ 3 (a) $6.63 \times 10^{-16} \text{ J}$.

According to Equation 3 ($E = hf$), the energy E of each photon is

$$E = (6.63 \times 10^{-34} \text{ J s}) \times (10^{18} \text{ Hz})$$

$$= 6.63 \times 10^{-16} \text{ J}$$

(b) $2.2 \times 10^{-24} \text{ kg m s}^{-1}$ in the photon's direction of motion.

According to Equation 5, the magnitude p of the photon's momentum is given by $p = E/c$

$$p = 6.63 \times 10^{-16} \text{ J} / (3.00 \times 10^8 \text{ m s}^{-1})$$

$$\approx 2.2 \times 10^{-24} \text{ kg m s}^{-1}$$

using the answer to part (a). Remember that the momentum of the photon is specified by a magnitude *and* a direction (the direction is the same as the photon's direction of motion).

SAQ 4 Evidence for the wave model of electrons is provided by the experiments on *electron diffraction*. Observations of the *Compton effect* provide evidence for the particle model of electromagnetic radiation.

SAQ 5 Statements (b) and (c) are correct: quantum mechanics describes the interactions and propagation of electrons (and all other matter). Statement (a) is false because quantum mechanics does not describe visible light, or any other form of electromagnetic radiation. Statement (d) is false because quantum mechanics applies to *all* matter.

SAQ 6 (a) An individual proton is most likely to be detected around the straight-through position (zero diffraction angle) because it is there that the pattern has its most pronounced diffraction maximum.

(b) An individual proton will definitely *not* be detected at the diffraction minima, where the intensities are zero. There is *zero* probability of detecting a proton at each of these minima.

(c) The probability of detecting an individual proton with a diffraction angle of 2.2° is approximately half the probability of detecting it around the straight-through position, because the intensity when the diffraction angle is 2.2° is approximately half the intensity when the diffraction angle is zero degrees.

SAQ 7 (a) See Figure 41a, an infinite sine wave; (b) see Figure 41b, a standing wave. Note that you could have drawn *any* infinite sine wave to answer part (a) and *any* standing wave to answer part (b).

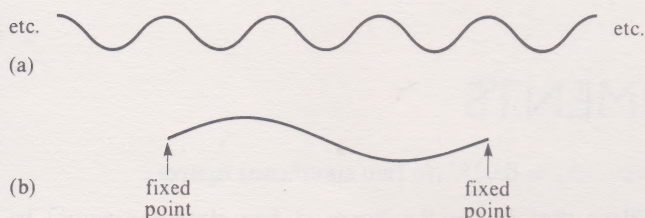


FIGURE 41 See the answer to SAQ 7.

SAQ 8 $3.6 \times 10^{-10} \text{ m}$.

The wavefunction of a free quantum is an infinite sine wave (Frame 3 of the AV sequence) whose wavelength is given by the de Broglie equation $\lambda_{\text{dB}} = h/mv$ (Equation 8). For an electron with the speed specified in this question

$$\lambda_{\text{dB}} = \frac{6.63 \times 10^{-34} \text{ J s}}{(9.11 \times 10^{-31} \text{ kg}) \times (2.00 \times 10^6 \text{ m s}^{-1})}$$

i.e. $\lambda_{\text{dB}} = 3.6 \times 10^{-10} \text{ m}$

SAQ 9 (a) Zero, because the value of the wavefunction is zero at this position. Remember from Frame 4 of the AV sequence that the probability of detecting a quantum of matter in a given small region of space is proportional to the square of the value of the wavefunction in that region.

(b) The probability of detecting the quantum at A is the same as the probability of its detection at B, because the *magnitude* of the value of the wavefunction is the same at both points, so

$$\left(\begin{array}{c} \text{value of the} \\ \text{wavefunction at A} \end{array} \right)^2 = \left(\begin{array}{c} \text{value of the} \\ \text{wavefunction at B} \end{array} \right)^2$$

(c) The probability of detecting the quantum at C is greater than the probability of its detection at D because the magnitude of the value of the wavefunction at C is greater than the magnitude of the value of the wavefunction at D. Hence,

$$\left(\begin{array}{c} \text{value of the} \\ \text{wavefunction at C} \end{array} \right)^2 \text{ is greater than } \left(\begin{array}{c} \text{value of the} \\ \text{wavefunction at D} \end{array} \right)^2$$

SAQ 10 Given the degree of superficiality of the statement, it would be reasonable of you to explain the meaning of the principle in terms that do not involve components of position and momentum.

Your non-mathematical summary of the principle should go something like this: basically, it tells us about the accuracies with which position and momentum can be specified *at the same time* (the position of course specifies where something is and its momentum depends on its mass, on how quickly it is moving and on its direction of motion.) The principle tells you how to calculate the uncertainty in one of the quantities, given the uncertainty in the other quantity *at the same time*: the product of the two is *at least* equal to a tiny fixed quantity (Planck's constant divided by 4π). This implies that if one variable is known very accurately then the other variable must *at the same time* be known very inaccurately.

Note that, according to the principle, it is quite possible for *either* the position *or* the momentum to be known for certain. However, the principle says that if one of these quantities *is* known for certain, then it follows that the corresponding quantity *at the same time* is indeed infinitely uncertain. All this is, of course, a very far cry from the erroneous simplicities of the amateur philosopher's original statement!

SAQ 11 (a) $\Delta v_x = \Delta v_y = \Delta v_z \approx 6.3 \times 10^6 \text{ m s}^{-1}$.

The uncertainty Δp_x in the x -component of momentum of a quantum in the nucleus is, according to Heisenberg's uncertainty principle,

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

where Δx is the corresponding uncertainty in x and h is Planck's constant (precisely analogous formulae apply to the y - and z -components of position and momentum). The uncertainty Δx may be taken to be $5 \times 10^{-15} \text{ m}$, half the diameter of the nucleus. Hence,

$$(5.00 \times 10^{-15} \text{ m}) \times \Delta p_x \geq \frac{6.63 \times 10^{-34} \text{ J s}}{4 \times 3.14}$$

i.e. $\Delta p_x \geq 1.06 \times 10^{-20} \text{ kg m s}^{-1}$

The corresponding uncertainty Δv_x in the x -component of velocity is given by

$$\Delta p_x = m \Delta v_x$$

where m may be taken as the average mass of the proton and neutron (i.e. $1.67 \times 10^{-27} \text{ kg}$)

$$\Delta v_x \geq \frac{1.06 \times 10^{-20} \text{ kg m s}^{-1}}{1.67 \times 10^{-27} \text{ kg}}$$

i.e. $\Delta v_x \geq 6.3 \times 10^6 \text{ m s}^{-1}$, 6.3 million metres per second.

By the same reasoning, Δv_y and Δv_z are also each at least $6.3 \times 10^6 \text{ m s}^{-1}$.

(b) The helium nucleus (and all other nuclei) should be visualized as a cloud whose fuzziness indicates the uncertainties in the velocity components of the constitu-

ents. Hence, an atom is visualized as an electron cloud (of diameter approximately 5×10^{-10} m) with a comparatively tiny nuclear cloud (of diameter approximately 10^{-14} m) at its centre.

SAQ 12 If there were a state of a nucleus in which a constituent proton or neutron were permanently at rest, the uncertainty in each of the momentum components of the particle would be zero so, according to the uncertainty principle, the uncertainty in the particle's position components would be infinite. This is not the case, because protons and neutrons in nuclei are normally confined to a tiny region of space (the typical nuclear diameter is approximately 10^{-14} m). Hence, there cannot exist a nuclear state in which a constituent proton or neutron is permanently at rest.

This explanation is precisely analogous to the explanation given in the text of why there is no state of an atom in which a constituent electron is permanently at rest.

SAQ 13 Given that the momentum of a free quantum is known for certain, it follows from the uncertainty principle that the position of the quantum will be infi-

nitely uncertain. Hence, you should expect that the wavefunction of the free quantum (which, as you saw in the AV sequence, gives information about the probability of detecting it in different regions) should be of infinite extent to reflect this uncertainty in its position.

SAQ 14 (i) Each of the situations will be accounted for by quantum mechanics, which applies to *all* matter, microscopic and macroscopic.

(ii) Newtonian mechanics can be applied reliably *only* to (c) and (d), i.e. to macroscopic situations.

SAQ 15 Statement 1 is wrong because the walker will be diffracted, and the diffraction will depend on the width of the doorway, just as the diffraction of an electron by a single slit depends on the width of the slit (Section 4.2). Statement 2 is wrong because the path of the stone, which will be diffracted as it enters the well, cannot be predicted for certain just as the path of a diffracted electron cannot be predicted for certain (Section 4.2).

Note that the quantum effects specified in this answer are absolutely minute—much too small to be observed in practice.

ACKNOWLEDGEMENTS

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